

SOLUTION #8 (9 AM)

Solution 1. (1) $Q(0) = A(1 - e^0) = A(1 - 1) = A \cdot 0 = 0$

(2) $Q'(t) = A(-1 \cdot (-k)e^{-kt}) = kAe^{-kt}$

(3) $\lim_{t \rightarrow \infty} Q(t) = \lim_{t \rightarrow \infty} A(1 - e^{-kt}) = A(1 - \lim_{t \rightarrow \infty} \frac{1}{e^{kt}}) = A \cdot (1 - 0) = A$

(4) $A = 10$

$$A(15) = 5 \implies 10(1 - e^{-k \cdot 15}) = 5 \implies 1 - e^{-15kt} = \frac{5}{10} = \frac{1}{2}$$
$$\implies e^{-15kt} = \frac{1}{2} \implies -15k = \ln\left(\frac{1}{2}\right) = -\ln 2 \implies k = \frac{\ln 2}{15}$$

Solution 2. (1) $f'(x) = 3x^2 \cdot e^{2x} + x^3 \cdot 2 \cdot e^{2x} = (3x^2 + 2x^3)e^{2x}$

(2) $g'(x) = 2 \ln x^2 \cdot (\ln x^2)' = 2 \ln x^2 \cdot \frac{2x}{x^2} = \frac{4 \ln x^2}{x}$

OR

Notice $g(x) = (2 \ln x)^2 = 4(\ln x)^2$, so

$g'(x) = 4 \cdot 2 \ln x (\ln x)' = 8 \ln x \cdot \frac{1}{x} = \frac{8 \ln x}{x}$