

Math 402: Spring 2008 Practice Test I

- (a) State both parts of the Incidence Axiom.
(b) For each of the following five spaces, decide whether the Incidence Axiom (both parts) is true or not. For each space, explain why it is true or give a counterexample to show that it is not. Also, state any relevant theorems.

Spaces:

E^2 , S^2 , H^2 , **the cylinder**, **the plane with a disk removed**

- On the cylinder, are **(a) helices** and **(b) intersections of the cylinder with a tilted plane (neither horizontal nor vertical)** geodesics or not? State as many reasons for each answer as you can.
- Given a point P in H^2 and an angle measure $\theta > 0$, is there always a rotation around P by θ , or not? Explain why or why not. Make sure your explanation is clear and complete enough to convince a skeptic.
- Define isometry. Is the composition of two isometries always an isometry. If no, find a counterexamples. If yes, give a proof.
- Let P be a point on a line l in E^2 , S^2 or H^2 . Prove that there is a unique line m through P so that reflection through m takes l to itself. Make sure that your proof works for each of the three spaces.