

Journal on Congruence.

Begin by considering two point a and p . On the Euclidean plane, can you always find an isometry which takes a to p ? If so, which isometry? Can you always find a single reflection which takes a to p ? Is so, reflection across what line? Will the same procedures work on the sphere? Will they work on hyperbolic space? What is true on these spaces? Try to prove your claims. Are there any gaps in your proof?

Hint: to do these problems, use your overheads. For example, you could write the letter a on a piece of paper and the letter p on the overhead. Then try to find an isometry which matches them up.

Now consider two rays \overrightarrow{ab} and \overrightarrow{pq} . On the plane, can you always find an isometry (or a series of isometries) which takes \overrightarrow{ab} to \overrightarrow{pq} ? Can you always find a single reflection which takes \overrightarrow{ab} to \overrightarrow{cd} ? How about a series of reflections? Again, describe the isometry/reflection in each case. What happens when you attempt to repeat this argument on the sphere? On hyperbolic space? Again, try to see whether you can make this rigorous.

Go back and repeat the paragraph above with two segments, \overline{ab} and \overline{pq} . Does the same argument work? If not, does it work if you make some restrictions? Finally, consider what happens if you instead consider $\angle abc$ and $\angle pqr$.