

Putnam Training Session 2

Tools: Series identities

1. **Geometric series.** $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ ($|x| < 1$)
2. **Finite geometric series.** $\sum_{k=0}^n x^k = \frac{1-x^{n+1}}{1-x}$ ($n = 1, 2, \dots, x \neq 1$)
3. **Exponential series.** $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$
4. **Logarithmic series.** $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = \log(1+x)$ ($|x| < 1$)
5. **Binomial theorem.** $\sum_{k=0}^n \binom{n}{k} x^k = (1+x)^n$ ($n = 1, 2, \dots, x$ real)
6. **Binomial series.** $\sum_{k=0}^{\infty} \binom{\alpha}{k} x^k = (1+x)^\alpha$ ($|x| < 1, \alpha$ any real number),
where $\binom{\alpha}{k} = (\alpha)(\alpha-1)\cdots(\alpha-k+1)/k!$.

Problem Set 3: Infinite series

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.
2. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.
3. $\sum_{n=k}^{\infty} x^n$ ($k = 0, 1, 2, \dots, |x| < 1$)
4. $\sum_{n=0}^{\infty} \frac{n}{2^n}$
5. $\sum_{n=1}^{\infty} \frac{1}{n2^n}$
6. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$
7. $\sum_{n=0}^{\infty} \frac{(n+1)^2}{n!}$
8. $\sum_{n=0}^{\infty} \binom{n+k}{k} x^n$ ($k = 0, 1, 2, \dots, |x| < 1$)
9. $\sum_{n=1}^{\infty} \frac{(-1)^{\lfloor 2^n x \rfloor}}{2^n}$ ($0 < x < 1$) ($\lfloor t \rfloor$ denotes the greatest integer $\leq t$)
10. $\sum_{n=1}^{\infty} \frac{s(n)}{n(n+1)}$, where $s(n)$ is the number of 1's in the binary expansion of n . (Putnam '81, B5)