

QUIZ2

Name : _____

Date: _____

This quiz will be used to check your attendance and performance. You can use your class notes and handouts. If you need, it is o.k. to discuss with your classmates.

1.

An actuary determines that the claim size for a certain class of accidents is a random variable, X , with moment generating function

$$M_X(t) = \frac{1}{(1 - 2500t)^4}.$$

Determine the standard deviation of the claim size for this class of accidents.

- (A) 1,340 (B) 5,000 (C) 8,660 (D) 10,000 (E) 11,180

Show your work in details.

Answer: B: 5,000

Solution: Use the formulas $M'(0) = E(X)$, $M''(0) = E(X^2)$.

2.

Let X be a random variable with moment generating function

$$M(t) = \left(\frac{2 + e^t}{3}\right)^9, \quad -\infty < t < +\infty.$$

Calculate the variance of X .

- (A) 2 (B) 3 (C) 8 (D) 9 (E) 11

Show your work in details.

Answer: A: 2

Solution: Use the formulas $E(X) = M'(0)$ and $E(X^2) = M''(0)$. Be careful when computing the derivatives of $M(t)$, using the chain rule!

3.

A company prices its hurricane insurance using the following assumptions:

- (i) In any calendar year, there can be at most one hurricane.
- (ii) In any calendar year, the probability of a hurricane is 0.05.
- (iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

- (A) 0.06 (B) 0.19 (C) 0.38 (D) 0.62 (E) 0.92

Answer: E: 0.92

Solution: By the given assumptions the occurrence of hurricanes can be modeled as success/failure trials, with success meaning that a hurricane occurs in a given year and occurring with probability $p = 0.05$. Thus, the probability that there are fewer than 3 hurricanes in a 20-year period is equal to the probability of having less than 3 successes in 20 success/failure trials with $p = 0.05$. By the binomial distribution, this probability is

$$\begin{aligned} & \sum_{x=0}^2 \binom{20}{x} 0.05^x 0.95^{20-x} \\ & = 0.95^{20} + 20 \cdot 0.05 \cdot 0.95^{19} + \frac{20 \cdot 19}{2} \cdot 0.05^2 \cdot 0.95^{18} = 0.92. \end{aligned}$$

4.

A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have Poisson distribution with mean 1.5. What is the expected amount paid to the company under this policy during a one-year period?

- (A) 2,769 (B) 5,000 (C) 7,231 (D) 8,347 (E) 10,578

Show your work in details.

Answer: C: 7,231

Solution: The main difficulty in this problem is that it requires a slightly tricky manipulation of the exponential series. Let X denote the number of snowstorms, and let Y denote the insurance payout, measured in units of 10,000. We need to calculate $E(Y)$. From the given information, X is Poisson distributed with mean 1.5 and Y is related to X by

$$Y = \begin{cases} X - 1 & \text{if } X = 1, 2, \dots, \\ 0 & \text{if } X = 0. \end{cases}$$

Hence,

$$\begin{aligned} E(Y) &= \sum_{x=1}^{\infty} (x-1)P(X=x) = \sum_{x=1}^{\infty} (x-1)e^{-1.5} \frac{1.5^x}{x!} \\ &= e^{-1.5} \left(\sum_{x=1}^{\infty} x \frac{1.5^x}{x!} - \sum_{x=1}^{\infty} \frac{1.5^x}{x!} \right) \\ &= e^{-1.5} \left(1.5 \sum_{x=1}^{\infty} \frac{1.5^{x-1}}{(x-1)!} - \left(\sum_{x=0}^{\infty} \frac{1.5^x}{x!} - 1 \right) \right) \\ &= e^{-1.5} (1.5e^{1.5} - e^{1.5} + 1) \\ &= 0.5 - e^{-1.5} = 0.7231. \end{aligned}$$

Since the units of Y are 10,000, we get 7,231 as answer.

5.

A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?

- (A) 0.096 (B) 0.192 (C) 0.235 (D) 0.376 (E) 0.469

Show your work in details.

Answer: E: 0.469

Solution: This is a two stage problem. For the first stage, focus on a single group, and consider the probability, say q , that in this group at least 9 participants complete the study. Then q is the probability that in 10 success/failure trials with success probability $p = 0.8$ there are at least 9 successes, so q can be computed using the binomial distribution: $q = 1 - 0.8^{10} - \binom{10}{9}0.8^9 0.2 = 0.62419$.

For the second stage, we consider two such groups, and compute the probability that exactly one of them is “successful” in the above sense. This probability is given by $2q(1-q) = 0.46939$, as can be seen either by considering separately the cases “group 1 successful, group 2 unsuccessful” and “group 1 unsuccessful, group 2 successful”, or by another application of the success/trial model, this time with 2 trials and success probability q .