QUIZ 3

Name : **Date:**

This quiz will be used to check your attendance and performance. You can use your class notes and handouts. If you need, it is o.k. to discuss with your classmates.

1.

An insurance company issued insurance policies to 32 independent risks. For each policy, the probability of a claim is 1/6. The benefit given that there is a claim has probability density function

$$f(y) = \begin{cases} 2(1-y), & 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of total benefits paid.

(A) $\frac{16}{9}$ (B) $\frac{8}{3}$ (C) $\frac{32}{9}$ (D) $\frac{16}{3}$ (E) $\frac{32}{3}$

Show your work in details.

Answer: A: 16/9

Hint/Solution: The expected value of a single benefit is $E(Y) = \int_0^1 yf(y)dy = \int_0^1 (2y - 2y^2)dy = 1/3$. The expected number of claims is $32 \cdot (1/6) = 16/3$. Multiplying these two numbers gives the expected value of all benefits paid, $(1/3)(16/3) = 16/9$.

2.

A large company has determined that the function

$$f(x) = 3x^2, \quad 0 \leq x \leq 1,$$

serves as the payroll density function. That is, the distribution of payroll, $F(x)$, which is the proportion of total payroll earned by the lowest paid fraction x of employees, $0 \leq x \leq 1$, relates to $f(x)$ in the same way that probability distributions and probability densities relate. Gini's index, G, defined as

$$G = 2\int_0^1 |x - F(x)|dx$$

is a measure of how evenly payroll is distributed among all employees. Calculate G for this large company.

(A) 0.2 (B) 0.4 (C) 0.5 (D) 0.8 (E) 1.0

Show your work in details.

Answer: C: 0.5

Hint/Solution: This reduces to an easy integration problem. First compute $F(x)$ by integrating $f(x)$, then compute the integral in the definition of G.

3.

An insurance company's monthly claims are modeled by a continuous, positive random variable X, whose probability density function is proportional to $(1+x)^{-4}$, where $0 < x < \infty$. Determine the company's expected monthly claims.

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) 3

Show your work in details.

Answer: C: 1/2

Hint/Solution: We have $f(x) = c(1+x)^{-4}$ for $0 < x < \infty$, where c is a constant. First determine the proportionality constant c by setting $\int_0^\infty c(1+x)^{-4}$ equal to 1 and solving for c. This gives $c = 3$. Then compute the integral $E(X) = \int_0^\infty xf(x)dx = \int_0^\infty 3x(1+x)^{-4}dx$ by substituting $u = 1+x$, $du = dx$.

4.

An insurance policy reimburses a loss up to a benefit limit of 10. The policyholder's loss, Y, follows a distribution with density function

$$f(y) = \begin{cases} 2y^{-3} & \text{for } y > 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected value of the benefit paid under the insurance policy?

(A) 1.0 (B) 1.3 (C) 1.8 (D) 1.9 (E) 2.0

Show your work in details.

Answer: D: 1.9

Hint/Solution: The benefit, X, is given by $X = Y$ if $Y \leq 10$, and by $X = 10$ if $Y > 10$. Thus,

$$E(X) = \int_1^{10} y \cdot 2y^{-3}dy + \int_{10}^{\infty} 10 \cdot 2y^{-3}dy = 2\left(1 - \frac{1}{10}\right) + 10 \cdot 10^{-2} = 1.9$$

5.

An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100. Losses incurred follow an exponential distribution with mean 300. What is the 95th percentile of actual losses that exceed the deductible?

(A) 600 (B) 700 (C) 800 (D) 900 (E) 1000

Show your work in details.

Hint/Solution: The main difficulty here is the correct interpretation of the "95th percentile of actual losses that exceed the deductible". The proper interpretation involves a conditional probability: we seek the value x such that the conditional probability that the loss is at most x, given that it exceeds the deductible, is 0.95, i.e., that $P(X \leq x|X \geq 100) = 0.95$, where X denotes the loss. By the complement formula for conditional probabilities, this is equivalent to $P(X \geq x|X \geq 100) = 0.05$. Since X is exponentially distributed with mean 300, we have $P(X \geq x) = e^{-x/300}$, so for $x > 100$,

$$P(X \geq x|X \geq 100) = \frac{P(X \geq x)}{P(X \geq 100} = \frac{e^{-x/300}}{e^{-100/300}} = e^{-(x-100)/300}.$$

Setting this equal to 0.05 and solving for x, we get $(x - 100)/300 = -\ln(0.05)$, so $x = -300\ln(0.05) + 100 = 1000$.