QUIZ4

Name : **Date:**

This quiz will be used to check your attendance and performance. You can use your class notes and handouts. If you need, it is o.k. to discuss with your classmates.

1.

Under a group insurance policy, an insurer agrees to pay 100% of the medical bills incurred during the year by employees of a small company, up to a maximum total of one million dollars. The total amount of bills incurred, X, has probability density function

$$f(x) = \begin{cases} \frac{x(4-x)}{9} & \text{for } 0 < x < 3, \\ 0 & \text{otherwise,} \end{cases}$$

where x is measured in millions. Calculate the total amount, in millions of dollars, the insurer would expect to pay under this policy.

(A) 0.120 (B) 0.301 (C) 0.935 (D) 2.338 (E) 3.495

Show your work in details.

Answer: C: 0.935

Hint/Solution: Let Y denote the payout (in millions). Then $Y = X$ if $X \leq 1$, and $Y = 1$ otherwise. Thus,

$$\begin{aligned} E(Y) &= \int_0^1 x f(x) dx + \int_1^\infty 1 \cdot f(x) dx \\ &= \int_0^1 \frac{1}{9} x^2 (4-x) dx + \int_1^3 \frac{1}{9} x(4-x) dx \\ &= \frac{1}{9}\left(4\frac{1}{3} - \frac{1}{4}\right) + \frac{1}{9}\left(\frac{3^2-1^2}{2} - \frac{3^3-1^3}{3}\right) \\ &= \frac{13}{108} + \frac{22}{27} = 0.935. \end{aligned}$$

2.

The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

(A) 6,321 (B) 7,358 (C) 7,869 (D) 10,256 (E) 12,642

Show your work in details.

Hint/Solution: Let X denote the lifetime of the printer. Then the refund for each printer, Y, is given by $Y = 200$ if $X \leq 1$, $Y = 100$ if $1 \leq X \leq 2$, and $Y = 0$ otherwise. Thus, $E(Y) = 200P(X \leq 1) + 100P(1 < X \leq 2)$. The probabilities $P(\dots)$ here are easily computed using that fact X has exponential distribution with mean 2, and thus c.d.f. $F(x) = P(X \leq x) = 1 - e^{-x/2}$ for $x \geq 0$. Thus, $P(X \leq 1) = 1 - e^{-1/2}$, $P(1 < X \leq 2) = P(X \leq 2) - P(X \leq 1) = (1 - e^{-2/2}) - (1 - e^{-1/2}) = e^{-1/2} - e^{-1}$, and hence $E(Y) = 200 \cdot (1 - e^{-1/2}) + 100 \cdot (e^{-1/2} - e^{-1}) = 102.5$. Multiplying by the number of printers, 100, gives the total refund, 10,250.

3.

Let X be exponentially distributed with mean 2. Determine:

$P(2 \leq X \leq 5)$.

$P(X \leq 5 | X \geq 2)$.

Show your work in details.

Let X be exponentially distributed with mean 2. Determine:

$P(2 \leq X \leq 5)$.

Solution: We have $F(x) = 1 - e^{-x/2}$ for $x \geq 0$, so $P(2 \leq X \leq 5) = F(5) - F(2) = (1 - e^{-5/2}) - (1 - e^{-2/2}) = e^{-1} - e^{-5/2}$.

$P(X \geq 5 | X \geq 2)$.

Solution: By the definition of conditional probabilities,

$$P(X \geq 5 | X \geq 2) = \frac{P(X \geq 5 \text{ and } X \geq 2)}{P(X \geq 2)}$$
$$= \frac{P(X \geq 5)}{P(X \geq 2)} = \frac{e^{-5/2}}{e^{-2/2}} = e^{-3/2}.$$

4.

Assume the amount of damage, X, in an auto accident is exponentially distributed with mean 2. (All figures are thousands of dollars.)

Suppose the insurance company covers the full amount of the loss up to 1, and 50% of any loss in excess of 1. What is the average payoff?

Show your work in details.

Suppose the insurance company covers the full amount of the loss up to 1, and 50% of any loss in excess of 1. What is the average payoff?

Solution: Letting Y denote the payoff, we now have

$$Y = \begin{cases} X & \text{if } X \leq 1, \\ 1 + (1/2)(X-1) = (1/2)(X+1) & \text{if } X > 1. \end{cases}$$

We need to compute $E(Y)$. By the calculation of Problem 7, we get $E(Y) = 2(1 - \frac{1}{2}e^{-1/2})$.