QUIZ4

Name : **Date:**

This quiz will be used to check your attendance and performance. You can use your class notes and handouts. If you need, it is o.k. to discuss with your classmates.

1.

Let X and Y be discrete random variables with joint probability function

$$p(x,y) = \begin{cases} \frac{2x+y}{12} & \text{for } (x,y) = (0,1),(0,2),(1,2),(1,3), \\ 0 & \text{otherwise.} \end{cases}$$

Determine the marginal probability function of X.

(A) $p(x) = \begin{cases} 1/6 & \text{for } x = 0, \\ 5/6 & \text{for } x = 1, \\ 0 & \text{otherwise.} \end{cases}$

(B) $p(x) = \begin{cases} 1/4 & \text{for } x = 0, \\ 3/4 & \text{for } x = 1, \\ 0 & \text{otherwise.} \end{cases}$

(C) $p(x) = \begin{cases} 1/3 & \text{for } x = 0, \\ 2/3 & \text{for } x = 1, \\ 0 & \text{otherwise.} \end{cases}$

(D) $p(x) = \begin{cases} 2/9 & \text{for } x = 1, \\ 3/9 & \text{for } x = 2, \\ 4/9 & \text{for } x = 3, \\ 0 & \text{otherwise.} \end{cases}$

(E) $p(x) = \begin{cases} y/12 & \text{for } x = 0, \\ (2+y)/12 & \text{for } x = 1, \\ 0 & \text{otherwise.} \end{cases}$

Show your work in details.

Answer: B

Solution: This is a routine calculation of a marginal distribution.

2.

An actuary determines that the annual numbers of tornadoes in counties P and Q are jointly distributed as follows:

		Annual number of tornadoes in county Q			
		0	1	2	3
Annual number	0	0.12	0.06	0.05	0.02
of tornadoes	1	0.13	0.15	0.12	0.03
in county P	2	0.05	0.15	0.10	0.02

Calculate the conditional variance of the annual number of tornadoes in county Q, given that there are no tornadoes in county P.

(A) 0.51 (B) 0.84 (C) 0.88 (D) 0.99 (E) 1.76

Show your work in details.

Answer: D: 0.99

Solution: We need to compute $\text{Var}(Q|P=0)$, where P and Q denote, respectively, the number of tornadoes in counties P and Q. Now $\text{Var}(Q|P=0) = E(Q^2|P=0) - E(Q|P=0)^2$, and the two conditional expectations here are computed just like ordinary expectations, but with the conditional density of Q given $P=0$ in place of the ordinary density of Q. This conditional density is obtained from the first row of the distribution table (corresponding to $P=0$) by dividing by the row sum, $0.12+0.06+0.05+0.02 = 0.25$, resulting in values $0.48, 0.24, 0.20, 0.08$ for $Q = 0, 1, 2, 3$. Using these probabilities in computing the expectations $E(Q^2|P=0)$ and $E(Q|P=0)$ gives the result, 0.99.

3.

Let X and Y be random losses with joint density function

$$f(x, y) = e^{-x-y} \quad \text{for } x > 0 \text{ and } y > 0.$$

An insurance policy is written to reimburse $X+Y$. Calculate the probability that the reimbursement is less than 1.

(A) e^{-2} (B) e^{-1} (C) $1-e^{-1}$ (D) $1-2e^{-1}$ (E) $1-2e^{-2}$

Show your work in details.

Answer: D

Solution: The probability to compute is $P(X+Y \leq 1)$, which is given by the double integral of $f(x,y) = e^{-x-y}$ over the part of the first quadrant in which $x+y \leq 1$. A sketch shows that this region is a triangular region described by the inequalities $0 \leq x \leq 1, 0 \leq y \leq 1-x$. Therefore,

$$P(X+Y \leq 1) = \int_{x=0}^{1} \int_{y=0}^{1-x} e^{-x-y} dy dx = \int_{x=0}^{1} e^{-x}(1-e^{-(1-x)}) dx$$
$$= \int_{0}^{1} (e^{-x} - e^{-1}) dx = 1 - 2e^{-1}.$$

4.

A company has two electric generators. The time until failure for each generator follows an exponential distribution with mean 10. The company will begin using the second generator immediately after the first one fails. What is the variance of the total time that the generators produce electricity?

(A) 10 (B) 20 (C) 50 (D) 100 (E) 200

Show your work in details.

Answer: E: 200

Solution: We need to compute $\text{Var}(X+Y)$, where X and Y denote the times that the two generators run. Assuming X and Y are independent, we have $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$. Now X and Y are exponential with $\mu = \theta = 10$, so $\text{Var}(X) = \text{Var}(Y) = \theta^2 = 100$, and hence $\text{Var}(X+Y) = 200$.

Remark: The independence of X and Y here is not explicitly stated, but the only reasonable assumption given the context, and without this assumption the problem would not be doable. This is one of the rare cases (and, in fact, the only case I am aware of) in which an independence assumption is not explicitly stated in the problem itself, but has to be made in order for the problem to be solvable. In general, if there is no explicit independence assumption in a problem, chances are that independence does not hold, so you should not make any assumptions about independence.

5.

Let X represent the age of an insured automobile involved in an accident. Let Y represent the length of time the owner has insured the automobile at the time of the accident. X and Y have joint probability density function

$$f(x,y) = \begin{cases} \frac{1}{64}(10 - xy^2) & \text{for } 2 \leq x \leq 10 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected age of an insured automobile involved in an accident.

(A) 4.9 (B) 5.2 (C) 5.8 (D) 6.0 (E) 6.4

Show your work in details.

Answer: C: 5.8

Solution: We need to compute $E(X)$. This can be done either via double integrals, using the formula $E(X) = \int\int x f(x,y) dy dx$, or via a single integral, by computing first the marginal density $f_X(x)$ and then $E(X)$ via $E(X) = \int x f_X(x) dx$. Either approach is a straightforward, though somewhat lengthy, calculation and leads to 5.77 as answer.