Probability mass function

EX1.

1.

An insurance policy pays an individual 100 per day for up to 3 days of hospitalization and 25 per day for each day of hospitalization thereafter. The number of days of hospitalization, X, is a discrete random variable with probability function

$$P(X = k) = \begin{cases} \frac{6-k}{15} & \text{for } k = 1, 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected payment for hospitalization under this policy.

(A) 85 (B) 163 (C) 168 (D) 213 (E) 255

Answer: D: 213

Solution: A fairly easy exercise in discrete densities; just make a table of the *total* payment for each value of k, along with the associated probability $P(X = k)$, and then compute the expected value of this payment.

Expectation (mean) / Variance

EX2.

A probability distribution of the claim sizes for an auto insurance policy is given in the table below:

Claim size	Probability
20	0.15
30	0.10
40	0.05
50	0.20
60	0.10
70	0.10
80	0.30

What percentage of the claims are within one standard deviation of the mean claim size?

(A) 45% (B) 55% (C) 68% (D) 85% (E) 100%

Answer: A: 0.45

Solution: A routine, but rather lengthy computation: First find $\mu = E(X)$, $E(X^2)$, $\mathrm{Var}(X)$, and σ. Then add up the probabilities for those x-values that satisfy $|x - \mu| \le \sigma$.

Moment-generating function

EX3.

Let X_1, X_2, X_3 be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} \frac{1}{3} & \text{for } x = 0, \\ \frac{2}{3} & \text{for } x = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the moment generating function, $M(t)$, of $Y = X_1X_2X_3$.

(A) $\frac{19}{27} + \frac{8}{27}e^t$

(B) $1 + 2e^t$

(C) $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^3$

(D) $\frac{1}{27} + \frac{8}{27}e^{3t}$

(E) $\frac{1}{3} + \frac{2}{3}e^{3t}$

Answer: A

Solution: This is easier than it looks at first glance, since $Y = X_1X_2X_3$ takes on only values 0 and 1, and $Y = 1$ occurs if and only if all of X_1, X_2, X_3 are equal to 1. The latter occurs with probability $(2/3)^3 = 8/27$, $P(Y = 1) = 8/27$ and $P(Y = 0) = 1 - 8/27 = 19/27$, and therefore $M(t) = e^{0t}P(Y = 0) + e^{1t}P(Y = 1) = 1 \cdot (19/27) + e^t \cdot (8/27)$.

Binomial distribution b(n, p)

EX4.

A small commuter plane has 30 seats. The probability that any particular passenger will not show up for a flight is 0.10, independent of other passengers. The airline sells 32 tickets for the flight. Calculate the probability that more passengers show up for the flight than there are seats available.

(A) 0.0042 (B) 0.0343 (C) 0.0382 (D) 0.1221 (E) 0.1564

Answer: E: 0.1564

Solution: Consider the 32 tickets as 32 Bernoulli (success/failure) trials with success meaning that the passenger holding the ticket shows up (so that $p = 0.9$. The probability to compute is then that of getting 31 or 32 successes in 32 such trials, which is given by the binomial distribution.

Geometric distribution

EX5.

As part of the underwriting process for insurance, each prospective policyholder is tested for high blood pressure. Let X represent the number of tests completed when the first person with high blood pressure is found. The expected value of X is 12.5. Calculate the probability that the sixth person tested is the first one with high blood pressure.

(A) 0.000 (B) 0.053 (C) 0.080 (D) 0.316 (E) 0.394

Answer: B: 0.053

Solution: By the formulas for a geometric distribution, $\mu = 1/p$, so $p = 1/\mu = 1/12.5$, and $P(X = 6) = (1 - p)^5 p = (11.5/12.5)^5(1/12.5) = 0.053$.

Poisson distribution

EX6.

An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?

(A) $1/\sqrt{3}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

Answer: D: 2

Solution: Let X denote the number of claims. We are given that X has Poisson distribution and that $P(X = 2) = 3P(X = 4)$. Substitute the formula for a Poisson p.m.f., $P(X = x) = e^{-\lambda}\lambda^x/x!$ into this equation to determine λ. Then use the fact that $\sigma^2 = \lambda$ for a Poisson distribution with parameter λ.