EX1

Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{|x|}{10} & \text{for } -2 \le x \le 4, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected value of X.

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) 1 (D) $\frac{28}{15}$ (E) $\frac{12}{5}$

Answer: D: 28/15

Hint/Solution: This is an easy integration exercise. The only tricky part here is the correct handling of the absolute value $|x|$ appearing in the definition of $f(x)$: One has to split the integral into two parts, corresponding to ranges $-2 \le x \le 0$ and $0 < x \le 4$, replacing $|x|$ by $-x$ in the first range, and by x in the second range, before evaluating the integral.

EX2

The loss amount, X, for a medical insurance policy has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{9}\left(2x^2 - \frac{x^3}{3}\right), & 0 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

Calculate the mode of the distribution.

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{3}{2}$ (D) 2 (E) 3

Answer: D: 2

Hint/Solution: The mode of a distribution is the point where $f(x)$ is maximal. With the given distribution we have $f(x) = F'(x) = (4/9)x - (1/9)x^2$ for $0 < x < 3$, and $f(x) = 0$ outside the interval $[0, 3]$. Differentiating, we get $f'(x) = (4/9) - (2/9)x$, and setting this equal to 0, we see that $x = 2$ is the only critical point of f. Hence the mode must occur at $x = 2$.

EX3

An insurance policy reimburses dental expense, X, up to a maximum benefit of 250. The probability density function for X is

$$f(x) = \begin{cases} ce^{-0.004x} & \text{for } x \ge 0, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant. Calculate the median benefit for this policy.

(A) 161 (B) 165 (C) 173 (D) 182 (E) 250

Answer: C: 173

Hint/Solution: The given distribution has the form of an exponential distribution, so the constant c must be 0.004, and the c.d.f. is equal to $F(x) = 1 - e^{-0.004x}$. To obtain the median, set $F(x) = 0.5$ and solve for x: $1 - e^{-0.004x} = 0.5$, so $-0.004x = \ln 0.5$, and $x = -\ln(0.5)/0.004 = 173$.

EX4

The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year. What portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year?

(A) 0.15 (B) 0.34 (C) 0.43 (D) 0.57 (E) 0.66

Answer: C: 0.43

Hint/Solution: If X denotes the number of days elapsed until an accident occurs, then we are given that X is exponentially distributed, and that $P(X \le 50) = 0.3$, and we have to compute $P(X \le 80)$. By the general of the c.d.f. of an exponential distribution, we have $F(x) = P(X \le x) = 1 - e^{-x/\theta}$ for $x > 0$. By the given information, $F(50) = P(X \le 50) = 0.3$, which allows us to determine θ: We have $1 - e^{-50/\theta} = 0.3$, so $-50/\theta = \ln 0.7$, or $\theta = -50/\ln 0.7$. Substituting this back into $F(x)$ and setting $x = 80$, we get $F(80) = P(X \le 80) = 1 - e^{-80/\theta} = 1 - e^{(80/50)\ln 0.7} = 0.43$

EX5

An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} 3x^{-4} & \text{for } x > 1, \\ 0 & \text{otherwise.} \end{cases}$$

Given that a randomly selected home is insured for at least 1.5, what is the probability that it is insured for less than 2?

(A) 0.578 (B) 0.684 (C) 0.704 (D) 0.829 (E) 0.875

Answer: A: 0.578

Hint/Solution: We need to compute

$$P(X \leq 2 | X \geq 1.5) = \frac{P(1.5 \leq X \leq 2)}{P(X > 1.5)}.$$

Now,

$$P(X > 1.5) = \int_{1.5}^{\infty} 3x^{-4} = 1.5^{-3} = 1.5^{-3} = 0.296,$$

and

$$P(1.5 \leq X \leq 2) = P(X > 1.5) - P(X > 2) = 1.5^{-3} - 2^{-3} = 0.171,$$

so $P(X \leq 2 | X \geq 1.5) = 0.171/0.296 = 0.578$.

EX6

An insurance policy is written to cover a loss, X, where X has a uniform distribution on $[0, 1000]$. At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

(A) 250 (B) 375 (C) 500 (D) 625 (E) 750

Hint/Solution: First note, that without the deductible the expected payment would be 500 (since it is then equal to the loss, and the loss X is uniformly distributed on $[0, 1000]$). Next, let D denote the (unknown) deductible and Y the payment under deductible D. Then $Y = X - D$ if $D \leq X \leq 1000$, and $Y = 0$ if $X \leq D$. Thus,

$$E(Y) = \int_D^{1000} (x - D) f(x) dx = \int_D^{1000} (x - D) \frac{1}{1000} dx = \frac{1}{1000} \cdot \frac{(1000 - D)^2}{2} = \frac{(1000 - D)^2}{2000}$$

Now set this expression equal to 25% of 500, i.e., 125, and solve for D: $(1000 - D)^2/2000 = 125$, or $(1000 - D)^2 = 250000$, so $D = 500$.