

Practice Problems on Integrals

1. Evaluate the following integrals:

$$\int_0^1 (x^3 + 2x^5 + 3x^{10}) dx$$

$$\int_0^\infty x(1+x)^{-5} dx$$

$$\int_1^\infty xe^{-3x} dx$$

2. Given that X has density (p.d.f.)

$$f(x) = \begin{cases} 1 - |x| & \text{for } -1 < x < 1, \\ 0 & \text{otherwise,} \end{cases}$$

evaluate:

$$P(X \geq 1/2)$$

$$E(X)$$

$$F(x) \text{ (the c.d.f.)}$$

3. Let X be exponentially distributed with mean 2. Determine:

$$P(X \geq 5)$$

$$P(2 < X < 5)$$

$$P(X \leq 5 | X \geq 2)$$

4. Suppose X has exponential distribution with median 3. Determine:

(a) $E(X)$.

(b) The 75-th percentile of the distribution of X .

5. Let X be exponentially distributed with mean 2, and let Y be defined by

$$Y = \begin{cases} 0 & \text{if } X \leq 1, \\ X - 1 & \text{if } X > 1. \end{cases}$$

Find $E(Y)$.

6. Let X be exponentially distributed with mean 2, and let

$$Y = \begin{cases} X & \text{if } X \leq 5, \\ 5 & \text{if } X > 5. \end{cases}$$

Find $E(Y)$.

7. Let X be exponentially distributed with mean 2, and let Y be defined by

$$Y = \begin{cases} X & \text{if } X \leq 1, \\ (1/2)(X + 1) & \text{if } X > 1. \end{cases}$$

Find $E(Y)$.

8. Let X be exponentially distributed with mean 3, and let $Y = \max(X, 2)$. Find $E(Y)$.

9. Assume the amount of damage, X , in an auto accident is exponentially distributed with mean 2. (All figures are thousands of dollars.)

Suppose first the insurance company covers the actual amount of the loss, up to a maximum of 5. What is the average payoff?

Suppose now the insurance company covers the full amount of the loss minus a deductible of 1. What is the average payoff?