

Name: Answer Key

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Actuarial Science Program
DEPARTMENT OF MATHEMATICS

Math 370 (Z)
 Exam 2/FM Preparation

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More Annuities and Perpetuities
Summary Quiz

- (1) Payments will be made to you at a continuous rate of 100 per year for the next five years. The effective rate of interest over this period is $i = 0.060$. Calculate the present value, at time $t = 0$, of this payment stream.

- (A) 415 (B) 434 (C) 459 (D) 472 (E) 498

$$PV = 100 \cdot \bar{a}_{\overline{5}|0.06} = 100 \left(\frac{1 - v^{5.06}}{\ln(1.06)} \right) = \underline{433.75}$$

$$\text{OR: } PV = \int_0^5 100 \cdot (1.06)^{-t} dt = \int_0^5 100 \cdot e^{\ln(1.06)^{-t}} dt = \dots$$

- (2) A series of 100 payments starts at 10 at the end of the first month and increases by 5 each month thereafter. The annual effective rate of interest is 8%. Calculate the present value of this series of payments.

- (A) 14,000 (B) 15,000 (C) 16,000 (D) 17,000 (E) 18,000

$$PV = 10 \cdot a_{\overline{100}|j} + 5 \left[\frac{a_{\overline{100}|j} - 100 \cdot v_j^{100}}{j} \right] = \underline{16,994.10}$$

(WHERE $j = (1.08)^{1/12} - 1 = .006434$)

- (3) The present value of a 20-year annuity-immediate, with a first payment of $20X$ at the end of the first year and with payments which decrease by X each year thereafter, is \$10,000. Assuming an annual effective interest rate of 7%, calculate X .

- (A) 74 (B) 84 (C) 94 (D) 104 (E) 114

$$10,000 = X \cdot (Da)_{\overline{20}|0.07} = X \cdot \left(\frac{20 - a_{\overline{20}|0.07}}{.07} \right)$$

$$\Rightarrow X = \underline{74.42}$$

$$\text{OR: } 10,000 = 20X \cdot a_{\overline{20}|} - X \left[\frac{a_{\overline{20}|} - 20 \cdot v^{20}}{.07} \right]$$