UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Actuarial Science Program
DEPARTMENT OF MATHEMATICS

Math 370 (Z) Prof. Rick Gorvett
Exam 2/FM Preparation

Basic Interest Theory Material
Review Problems

Topic A: Effective Interest Rates

$$\boxed{i_n = \frac{A(n) - A(n-1)}{A(n-1)} = \frac{I_n}{A(n-1)}}$$

1) Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns the same annual effective interest rate. The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X. Calculate X.

$A(t)\,(1+i)^s = A(t+s) \qquad I_{t+1} = i \cdot A(t)$

(A) 28.0
(B) 31.3
(C) 34.6
(D) 36.7
(E) 38.9

$$\left.\begin{array}{l}\text{BRUCE: } X = I_{11} = i \cdot A(10) = i \cdot [100(1+i)^{10}] \\ \text{ROBBIE: } X = I_{17} = i \cdot A(16) = i \cdot [50(1+i)^{16}]\end{array}\right\}\ \text{EQUAL}$$

$\Rightarrow 2 = (1+i)^6 \Rightarrow i = .122462$

$\Rightarrow X = \underline{\underline{38.879}}$

(2005 sample question # 27)

Topic B: Simple vs Compound Interest

$$\boxed{\begin{array}{l}\textit{Simple}:\ A(0) \times (1+it) = A(t) \\ \textit{Compound}:\ A(0) \times (1+i)^t = A(t)\end{array}}$$

2) Eric deposits 100 into a savings account at time 0, which pays interest at a nominal rate of i, compounded semiannually. Mike deposits 200 into a different savings account at time 0, which pays simple interest at an annual rate of i. Eric and Mike earn the same amount of interest during the last 6 months of the 8th year. Calculate i.

(A) 9.06%
(B) 9.26%
(C) 9.46%
(D) 9.66%
(E) 9.86%

ERIC: $i/2$ = HALF-YEAR EFFECTIVE RATE.

$I_{16} = \frac{i}{2} \cdot A(15) = \frac{i}{2} \cdot \left[100\left(1+\frac{i}{2}\right)^{15}\right].$

MIKE: SIMPLE INTEREST $\rightarrow$ SAME I FOR ANY PERIOD.

(2005 sample question # 3)

$\Rightarrow I_{16} = 200 \cdot i \left(\frac{1}{2}\right) = 200\left(\frac{i}{2}\right)$

$\Rightarrow \left(1+\frac{i}{2}\right)^{15} = 2 \Rightarrow i = \underline{\underline{.09459}}$

Topic C: Nominal Interest Rates

$$\boxed{1+i=\left[1+\frac{i^{(m)}}{m}\right]^m}$$

$\equiv i^{(2)}$

3) At a nominal interest rate of i convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year. Calculate i.

(A) 2.75%
(B) 2.77%
(C) 2.79%
(D) 2.81%
(E) 2.83%

$$\underset{(a)}{1000}\left(1+\frac{i}{2}\right)^4+\underset{(b)}{1500}\left(1+\frac{i}{2}\right)^2=\underset{(-c)}{2600} \quad \left(\text{DIVIDE THROUGH BY } 1000\right)$$

$$\Rightarrow \left(1+\frac{i}{2}\right)^2=\frac{-1.5\pm\sqrt{(1.5)^2+4(2.6)}}{2}=1.028342$$

(May 2005 Exam 2 / FM, Problem 13)

$$\Rightarrow 1+\frac{i}{2}=(1.028342)^{1/2}=1.014092$$

$$\Rightarrow i=\underline{\underline{.0281}}$$

Topic D: Rates of Discount

$$\boxed{d_n=\frac{I_n}{A(n)};\quad 1-d=\left[1-\frac{d^{(m)}}{m}\right]^m;\quad A(0)\times(1-d)^{-t}=A(t)}$$

4) Calculate the nominal rate of discount convertible monthly that is equivalent to a nominal rate of interest of 18.9% per year convertible monthly.

(A) 18.0%
(B) 18.3%
(C) 18.6%
(D) 18.9%
(E) 19.2%

$$1+i=\left[1+\frac{i^{(m)}}{m}\right]^m=\left[1-\frac{d^{(p)}}{p}\right]^{-p}$$

$$m=p=12 \qquad i^{(12)}=.189$$

$$\Rightarrow 1.206263=\left[1-\frac{d^{(12)}}{12}\right]^{-12}$$

(May 2005 Exam 2 / FM, Problem 19)

$$\Rightarrow d^{(12)}=\underline{\underline{.186069}}$$

5) Jeff deposits 10 into a fund today and 20 fifteen years later. Interest is credited at a nominal discount rate of d compounded quarterly for the first 10 years, and at a nominal interest rate of 6% compounded semiannually thereafter. The accumulated balance in the fund at the end of 30 years is 100. Calculate d.

(A) 4.33%
(B) 4.43%
(C) 4.53%
(D) 4.63%
(E) 4.73%

$$10\left(1-\frac{d}{4}\right)^{-40}\left(1+\frac{.06}{2}\right)^{2(20)}+20\left(1+\frac{.06}{2}\right)^{2(15)}=100$$

$$\Rightarrow d=\underline{\underline{.04532}}$$

(2005 sample question # 12)

Topic E: Present Value

$$A(0) = A(n) \times v^n = \frac{A(n)}{(1+i)^n}; \quad d = iv$$

6) David can receive one of the following two payment streams:
   (i) 100 at time 0, 200 at time n, and 300 at time $2n$
   (ii) 600 at time 10

   At an annual effective interest rate of i, the present values of the two streams are equal. Given $v^n = 0.76$, determine i.

$$100 \cdot v^0 + 200 \cdot \underset{.76}{v^n} + 300 \cdot \underset{(.76)^2}{v^{2n}} = 600 \cdot v^{10}$$

$$\Rightarrow v^{10} = .708800$$

$$\Rightarrow i = \underline{.03502}$$

(A) 3.5%
(B) 4.0%
(C) 4.5%
(D) 5.0%
(E) 5.5%

(2005 sample question # 20)

7) Mike receives cash flows of 100 today, 200 in one year, and 100 in two years. The present value of these cash flows is 364.46 at an annual effective rate of interest i. Calculate i.

$$364.46 = 100v^0 + 200v^1 + 100v^2$$

$$\Rightarrow 100v^2 + 200v^1 - 264.46 = 0$$

$$\Rightarrow v = \frac{-200 \pm \sqrt{40{,}000 + 105{,}784}}{200} = .909084$$

$$\Rightarrow \frac{1}{v} = 1.100 = 1+i$$

$$\Rightarrow i = \underline{.10} \text{ or } \underline{10\%}$$

(A) 10%
(B) 11%
(C) 12%
(D) 13%
(E) 14%

(May 2005 Exam 2 / FM, Problem 7)

8) A store is running a promotion during which customers have two options for payment. Option one is to pay 90% of the purchase price two months after the date of sale. Option two is to deduct X% off the purchase price and pay cash on the date of sale. A customer wishes to determine X such that he is indifferent between the two options when valuing them using an effective annual interest rate of 8%. Which of the following equations of value would the customer need to solve?

EQUAL PVs.

$$\left.\begin{array}{l} \text{OPTION 1: } PV = \dfrac{.90 \cdot P}{(1.08)^{1/6}} \\ \text{OPTION 2: } PV = \left(1 - \dfrac{X}{100}\right) \cdot P \end{array}\right\} \text{EQUAL.}$$

$$\Rightarrow \underline{\frac{.90}{(1.08)^{1/6}} = \left(1 - \frac{X}{100}\right)}$$

(A) $\left(\frac{X}{100}\right)\left(1+\frac{0.08}{6}\right) = 0.90$

(B) $\left(1-\frac{X}{100}\right)\left(1+\frac{0.08}{6}\right) = 0.90$

(C) $\left(\frac{X}{100}\right)(1.08)^{1/6} = 0.90$

(D) $\left(\frac{X}{100}\right)\left(\frac{1.08}{1.06}\right) = 0.90$

(E) $\underline{\left(1-\frac{X}{100}\right)(1.08)^{1/6} = 0.90}$

(May 2005 Exam 2 / FM, Problem 18)

Topic F: Force of Interest

$$\delta = \ln(1+i); \quad e^{\delta} = 1+i$$
$$\delta_t = \frac{A'(t)}{A(t)}; \quad A(t) = A(0) \times \exp \int_0^t \delta(r)\,dr$$

9) Bruce deposits 100 into a bank account. His account is credited interest at a nominal rate of interest of 4% convertible semiannually. At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at a force of interest of δ. After 7.25 years, the value of each account is the same. Calculate δ.

(A) 0.0388
(B) 0.0392
(C) 0.0396
(D) 0.0404
(E) 0.0414

BRUCE: $100 \cdot (1.02)^{14.5} = 133.2608$

PETER: $133.2608 = 100 \cdot (e^{\delta})^{7.25} \Rightarrow \delta = \underline{\underline{.039605}}$

(2005 sample question # 1)

10) At time 0, deposits of 10,000 are made into each of Fund X and Fund Y. Fund X accumulates at an annual effective interest rate of 5%. Fund Y accumulates at a simple interest rate of 8%. At time *t*, the forces of interest on the two funds are equal. At time *t*, the accumulated value of Fund Y is greater than the accumulated value of Fund X by *Z*. Determine *Z*.

A) 1625
B) 1687
C) 1697
D) 1711
E) 1721

$X: i = .05 \Rightarrow \delta_t = \ln(1.05)$

$Y: 10{,}000(1+.08t) = A_Y(t) \Rightarrow \delta_t = \dfrac{.08}{1+.08t}$ *

$\Rightarrow \ln(1.05) = \dfrac{.08}{1+.08t} \Rightarrow t = 7.9959$

$\Rightarrow A_Y(8) - A_X(8) = Z = \underline{\underline{1625}}.$

(Course 2 Sample Exam # 2, Problem 23)

$* \ \dfrac{A_Y'(t)}{A_Y(t)} = \dfrac{.08(10{,}000)}{10000(1+.08t)} = \dfrac{.08}{1+.08t}$

11) Ernie makes deposits of 100 at time 0, and *X* at time 3. The fund grows at a force of interest

$\delta_t = t^2/100, \ t > 0.$

The amount of interest earned from time 3 to time 6 is also *X*. Calculate *X*.

(A) 385
(B) 485
(C) 585
(D) 685
(E) 785

$A(3)^- = 100 \cdot e^{\int_0^3 \frac{t^2}{100}dt} = 100 \cdot e^{\frac{t^3}{300}\big|_0^3} = 109.417$

$X = A(6) - A(3)^+ = \underbrace{\underbrace{(109.417 + X)}_{A(3)^+} \cdot e^{\int_3^6 \frac{t^2}{100}dt}}_{A(6)} - \underbrace{(109.417 + X)}_{A(3)^+}$

(2005 sample question # 13)

$\Rightarrow X = \underline{\underline{784.59}}$