

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Actuarial Science Program
DEPARTMENT OF MATHEMATICS

Math 370 (Z)
 Exam 2/FM Preparation

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Basic Annuities and Perpetuities
Review Problems

Topic A: Annuities-Immediate

$$Ra_{\overline{n}|i} = Rv_i^1 + Rv_i^2 + \dots + Rv_i^n \Rightarrow a_{\overline{n}|i} = \frac{1-v^n}{i}$$

$$Rs_{\overline{n}|i} = R(1+i)^{n-1} + \dots + R(1+i)^0 \Rightarrow s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$$

$$s_{\overline{n}|i} = a_{\overline{n}|i}(1+i)^n \Rightarrow \frac{1}{a_{\overline{n}|i}} = \frac{1}{s_{\overline{n}|i}} + i$$

(1) Which of the following expressions does NOT represent a definition for $a_{\overline{n}|}$?

- (A) $v^n \left[\frac{(1+i)^n - 1}{i} \right] = v^n \cdot s_{\overline{n}|} = a_{\overline{n}|}$
- (B) $\frac{1-v^n}{i} = a_{\overline{n}|}$
- (C) $v + v^2 + \dots + v^n = a_{\overline{n}|} \Rightarrow$
- (D) $v \left[\frac{1-v^n}{1-v} \right]$
- (E) $\frac{s_{\overline{n}|}}{(1+i)^{n-1}}$
- $$A = v + v^2 + \dots + v^n$$

$$vA = v^2 + \dots + v^n + v^{n+1}$$

$$A(1-v) = v - v^{n+1} \Rightarrow A = \frac{v(1-v^n)}{1-v}$$

$$= \frac{v(1-v^n)}{\frac{1}{v} - 1} = \frac{1-v^n}{i}$$
- (May 2005 Course 2/FM Exam, Problem 1)
- Should Be $\frac{s_{\overline{n}|}}{(1+i)^n}$

- (2) To accumulate 8000 at the end of $3n$ years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the next $2n$ years. The annual effective rate of interest is i . You are given $(1+i)^n = 2.0$. Determine i .

(A) 11.25%
 (B) 11.75%
 (C) 12.25%
 (D) 12.75%
 (E) 13.25%

$$98 \cdot s_{\overline{n}|i} (1+i)^{2n} + 196 \cdot s_{\overline{2n}|i}$$

$$= 98 \frac{(1+i)^n - 1}{i} ((1+i)^n)^2 + 196 \frac{((1+i)^n)^2 - 1}{i} = 8000$$

(2005 sample question # 17) (November 2001 Course 2 Exam, # 12) } $i = \underline{\underline{.1225}}$

OR: $98 \cdot s_{\overline{3n}|} + 98 \cdot s_{\overline{2n}|} = 8000$

- (3) The parents of three children, ages 1, 3, and 6, wish to set up a trust fund that will pay X to each child upon attainment of age 18, and Y to each child upon attainment of age 21. They will establish the trust fund with a single investment of Z . Which of the following is the correct equation of value for Z ?

(A) $\frac{X}{v^{17} + v^{15} + v^{12}} + \frac{Y}{v^{20} + v^{18} + v^{15}}$
 (B) $3[Xv^{18} + Yv^{21}]$
 (C) $3Xv^3 + Y[v^{20} + v^{18} + v^{15}]$
 (D) $(X+Y) \frac{v^{20} + v^{18} + v^{15}}{v^3}$
 (E) $X[v^{17} + v^{15} + v^{12}] + Y[v^{20} + v^{18} + v^{15}]$

$$Z = X \cdot v^{18-1} + X \cdot v^{18-3} + X \cdot v^{18-6}$$

$$+ Y \cdot v^{21-1} + Y \cdot v^{21-3} + Y \cdot v^{21-6}$$

(Nov. 2005 Course 2/FM Exam, # 25)

- (4) For 10,000, Kelly purchases an annuity-immediate that pays 400 quarterly for the next 10 years. Calculate the annual nominal interest rate convertible monthly earned by Kelly's investment.

(A) 10.0%
 (B) 10.3%
 (C) 10.5%
 (D) 10.7%
 (E) 11.0%

$$10,000 = 400 \cdot a_{\overline{40}|j} \Rightarrow j = 2.524385$$

$$1+i = (1+j)^4 = 1.104864$$

(Nov. 2005 Course 2/FM Exam, Problem 13)

$$1+i = \left[1 + \frac{i^{(12)}}{12} \right]^{12} \Rightarrow i^{(12)} = \underline{\underline{.100137}}$$

- (5) Jim began saving money for his retirement by making monthly deposits of 200 into a fund earning 6% interest compounded monthly. The first deposit occurred on January 1, 1985. Jim became unemployed and missed making deposits 60 through 72. He then continued making monthly deposits of 200. How much did Jim accumulate in his fund on December 31, 1999?

- A) 53,572
 B) 53,715
 C) 53,840
 D) 53,966
 E) 54,184

$j = .06/12 = .005$

@ 12/31/99: $\begin{cases} \text{w/o MISSES: } 200 \cdot \ddot{s}_{\overline{180}|j} = 58,454.56 \\ \text{MISSES: } 200 \cdot \ddot{s}_{\overline{13}|j} \cdot (1+j)^{180-72} \\ \text{(May 2000 Course 2 Exam, Problem 47)} \end{cases} = 4,614.73$

$\Rightarrow \text{DIFF.} = \underline{\underline{53,839.83}}$

Topic B: Annuities-Due

$$R\ddot{a}_{\overline{n}|i} = Rv_i^0 + Rv_i^1 + \dots + Rv_i^{n-1} \Rightarrow \ddot{a}_{\overline{n}|i} = \frac{1-v^n}{d}$$

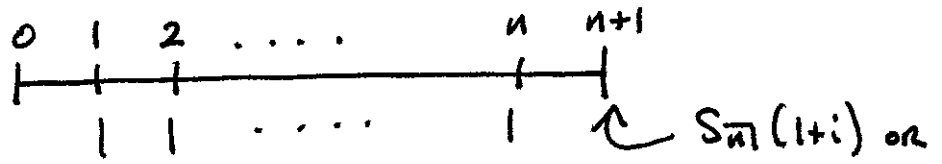
$$R\ddot{s}_{\overline{n}|i} = R(1+i)^n + \dots + R(1+i)^1 \Rightarrow \ddot{s}_{\overline{n}|i} = \frac{(1+i)^n - 1}{d}$$

$$\ddot{s}_{\overline{n}|i} = \ddot{a}_{\overline{n}|i} (1+i)^n \Rightarrow \frac{1}{\ddot{a}_{\overline{n}|i}} = \frac{1}{\ddot{s}_{\overline{n}|i}} + d$$

$$\ddot{a}_{\overline{n}|i} = a_{\overline{n}|i} (1+i) \Rightarrow \ddot{s}_{\overline{n}|i} = s_{\overline{n}|i} (1+i)$$

- (6) An annuity pays 1 at the end of each year for n years. Using an annual effective interest rate of i , the accumulated value of the annuity at time $(n+1)$ is 13.776. It is also known that $(1+i)^n = 2.476$. Calculate n .

- (A) 4
 (B) 5
 (C) 6
 (D) 7
 (E) 8
 24)



(May 2005 Course 2/FM Exam, Problem 24) $\ddot{s}_{\overline{n}|i} = 13.776$.

$13.776 = \frac{(1+i)^n - 1}{d} \Rightarrow d = .107143 = \frac{i}{1+i} (=iv)$

$\Rightarrow i = \frac{d}{1-d} = 0.1200$

$\Rightarrow (1.12)^n = 2.476$

$\Rightarrow n = \frac{\ln 2.476}{\ln 1.12} = \underline{\underline{8}}$

- (7) An investor accumulates a fund by making payments at the beginning of each month for 6 years. Her monthly payment is 50 for the first 2 years, 100 for the next 2 years, and 150 for the last 2 years. At the end of the 7th year the fund is worth 10,000. The annual effective interest rate is i , and the monthly effective interest rate is j . Which of the following formulas represents the equation of value for this fund accumulation?

(A) $s_{\overline{24}|i} (1+i) [(1+i)^4 + 2(1+i)^2 + 3] = 200$

(B) $s_{\overline{24}|j} (1+j) [(1+j)^4 + 2(1+j)^2 + 3] = 200$

(C) $s_{\overline{24}|i} (1+i) [(1+i)^4 + 2(1+i)^2 + 3] = 200$

(D) $s_{\overline{24}|j} (1+j) [(1+j)^4 + 2(1+j)^2 + 3] = 200$

(E) $s_{\overline{24}|j} (1+j) [(1+j)^4 + 2(1+j)^2 + 3] = 200$

$50 \cdot \ddot{s}_{\overline{24}|j} (1+i)^5 + 100 \cdot \ddot{s}_{\overline{24}|j} (1+i)^3 + 150 \cdot \ddot{s}_{\overline{24}|j} (1+i)^1 = 10,000$
 DIVIDE BY 50

(Nov. 2005 Course 2/FM Exam, # 3)

Topic C: Perpetuities

$$a_{\overline{\infty}|i} = \frac{1}{i} \Rightarrow \ddot{a}_{\overline{\infty}|i} = \frac{1}{d}$$

- (8) A perpetuity-immediate pays X per year. Brian receives the first n payments, Colleen receives the next n payments, and Jeff receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and Jeff's share is K . Calculate K .

(A) 24%
 (B) 28%
 (C) 32%
 (D) 36%
 (E) 40%

$B: .40 \left(\frac{X}{i} \right) = X \cdot a_{\overline{n}|i} \Rightarrow .40 = 1 - v^n \Rightarrow v^n = .60$
 $C: X \cdot a_{\overline{n}|i} \cdot v^n$

(2005 sample question # 25) (May 2001 Course 2 Exam, # 5)

$J: \left(\frac{X}{i} \right) \cdot v^{2n} = K \cdot \left(\frac{X}{i} \right) \Rightarrow K = v^{2n} = (v^n)^2$
 $= \underline{\underline{0.36}}$

- (9) An estate provides a perpetuity with payments of X at the end of each year. Seth, Susan, and Lori share the perpetuity such that Seth receives the payments of X for the first n years and Susan receives the payments of X for the next m years, after which Lori receives all the remaining payments of X . Which of the following represents the difference between the present value of Seth's and Susan's payments using a constant rate of interest?

(A) $X[a_{\overline{n}|} - v^n a_{\overline{m}|}]$

SETH: $PV = X \cdot a_{\overline{n}|}$

(B) $X[\ddot{a}_{\overline{n}|} - v^n \ddot{a}_{\overline{m}|}]$

SUSAN: $PV = X \cdot a_{\overline{m}|} \cdot v^n$

(C) $X[a_{\overline{n}|} - v^{n+1} a_{\overline{m}|}]$

(D) $X[a_{\overline{n}|} - v^{n-1} a_{\overline{m}|}]$

(E) $X[v a_{\overline{n}|} - v^{n+1} a_{\overline{m}|}]$

(May 2005 Course 2/FM Exam, Problem 4)

- (10) Which of the following are characteristics of all perpetuities?

- I. The present value is equal to the first payment divided by the annual effective interest rate.
 II. Payments continue forever.
 III. Each payment is equal to the interest earned on the principal.

INCLUDES:
 - IMMED. & DUE.
 - CONSTANT, GROWING, ...

NOT IF: (1) DUE (X/d).
 (2) NON-CONSTANT X.

- (A) I only
 (B) II only
 (C) III only
 (D) I, II, and III
 (E) The correct answer is not given by (A), (B), (C), or (D).

NOT FOR NON-CONSTANT X.

(May 2005 Course 2/FM Exam, Problem 12)