

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Actuarial Science Program
DEPARTMENT OF MATHEMATICS

Math 370 (Z)
 Exam 2/FM Preparation

Prof. Rick Gorvett

Rates of Return and Reinvestment Rates
Review Problems

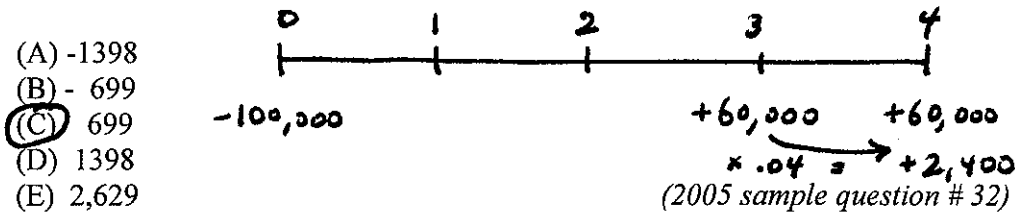
Topic A: Reinvestment Rates

$i = \text{original investment rate}; \quad j = \text{reinvestment rate}$

\$1 invested for n periods: $A(n) = 1 + is_{\overline{n}|j}$

\$1 - per - payment annuity - immediate: $A(n) = \frac{1}{j} + i(Is)_{\overline{n-1}|j} = n + i \left[\frac{s_{\overline{n}|j} - n}{j} \right]$

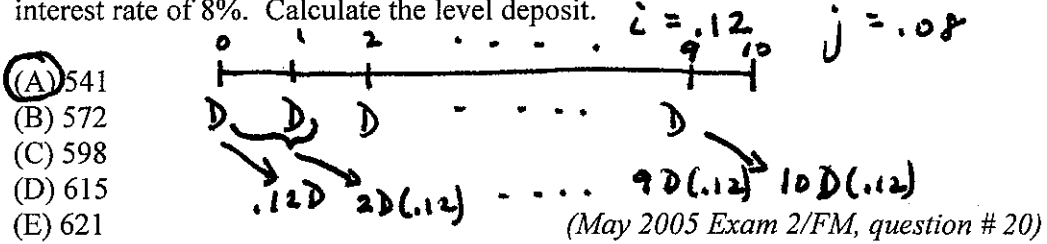
- (1) An investor pays \$100,000 today for a 4-year investment that returns cash flows of \$60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective. If the rate of interest at which the investment is to be valued is 5.0%, what is the net present value of this investment today?



$$A(4) = 60,000 + 60,000(1.04) = 122,400$$

$$NPV = \frac{122,400}{(1.05)^4} - 100,000 = \underline{\underline{698.79}}$$

- (2) An investor wishes to accumulate 10,000 at the end of 10 years by making level deposits at the beginning of each year. The deposits earn a 12% annual effective rate of interest paid at the end of each year. The interest is immediately reinvested at an annual effective interest rate of 8%. Calculate the level deposit.



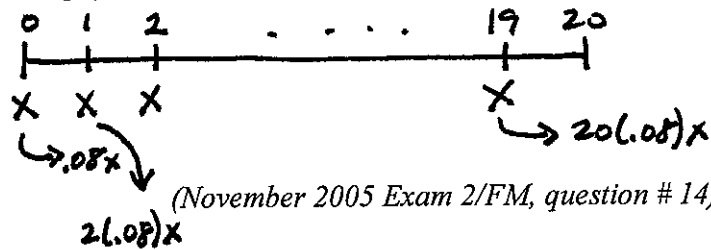
$$\Rightarrow 10,000 = 10D + \underbrace{.12D (Is)_{\overline{10}|.08}}_{.12D \left[\frac{\ddot{s}_{\overline{10}|.08} - 10}{.08} \right]} \Rightarrow D = \underline{\underline{541.47}}$$

8.468231D

- (3) Payments of X are made at the beginning of each year for 20 years. These payments earn interest at the end of each year at an annual effective rate of 8%. The interest is immediately reinvested at an annual effective rate of 6%. At the end of 20 years, the accumulated value of the 20 payments and the reinvested interest is 5600. Calculate X .

$i = .08$
 $j = .06$

- (A) 121.67
(B) 123.56
(C) 125.72
(D) 127.18
(E) 128.50



$$5600 = 20X + .08X \left(I \ddot{s} \right)_{\overline{20}|.06} \Rightarrow X = \underline{\underline{123.56}}$$

Topic B: Time-Weighted Rate of Return

$$(1+i) = (1+j_1) \cdots (1+j_m)$$

$j_k =$ return during k -th subperiod

Topic C: Dollar-Weighted Rate of Return

$$i \approx \frac{\text{"Withdrawals" - "Deposits"}}{\sum_t CF_t(1-t)}$$

$0 \leq t \leq 1$

- (4) You are given the following information about an investment account:

Date	Value Immediately Before Deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

TIME-WEIGHTED:

$$1 + .00 = \left(\frac{12}{10} \right) \left(\frac{X}{12+X} \right)$$

$$\Rightarrow X = 60$$

Over the year, the time-weighted return is 0%, and the dollar-weighted (money-weighted) return is Y . Calculate Y .

- (A) -25% (B) -10% (C) 0% (D) 10% (E) 25%

(2005 sample question # 45)

DOLLAR-WEIGHTED: $Y \approx \frac{X - 10 - X}{10(1) + X(\frac{1}{2})} = \frac{-10}{10 + 30} = -0.25$
OR -25%

- (5) You are given the following information about the activity in two different investment accounts:

Account K (D -WEIGHTED)			
Date	Fund value before activity	Activity	
		Deposit	Withdrawal
January 1, 1999	100.0		
July 1, 1999	125.0		X
October 1, 1999	110.0	2X	
December 31, 1999	125.0		

$$i_k \approx \frac{125 + X - 100 - 2X}{100(1) - X(\frac{1}{2}) + 2X(\frac{1}{4})} = \frac{25 - X}{100}$$

Account L (T IME-WEIGHTED)			
Date	Fund value before activity	Activity	
		Deposit	Withdrawal
January 1, 1999	100.0		
July 1, 1999	125.0		X
December 31, 1999	105.8		

$$1 + i_l = \left(\frac{125}{100}\right) \left(\frac{105.8}{125 - X}\right)$$

During 1999, the dollar-weighted (money-weighted) return for investment account K equals the time-weighted return for investment account L, which equals i . Calculate i .

- (A) 10% (B) 12% (C) 15% (D) 18% (E) 20%

(2005 sample question # 19) $1 + i_k = 1 + i_l \Rightarrow \dots X = 10 \Rightarrow i = \underline{\underline{0.15}}$

- (6) At the beginning of the year, an investment fund was established with an initial deposit of 1000. A new deposit of 1000 was made at the end of 4 months. Withdrawals of 200 and 500 were made at the end of 6 months and 8 months, respectively. The amount in the fund at the end of the year is 1560. Calculate the dollar-weighted (money-weighted) yield rate earned by the fund during the year.

- (A) 18.57%
 (B) 20.00%
 (C) 22.61%
 (D) 26.00%
 (E) 28.89%

$$i \approx \frac{1560 + 200 + 500 - 1000 - 1000}{1000(1) + 1000\left(\frac{8}{12}\right) - 200\left(\frac{6}{12}\right) - 500\left(\frac{4}{12}\right)}$$

(May 2005 Exam 2/FM, question # 16)

$$= \frac{260}{1400} = \underline{\underline{0.185714}}$$

- (7) An association had a fund balance of 75 on January 1 and 60 on December 31. At the end of every month during the year, the association deposited 10 from membership fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31. Calculate the dollar-weighted (money-weighted) rate of return for the year.

- (A) 9.0%
(B) 9.5%
(C) 10.0%
(D) 10.5%
(E) 11.0%

(2005 sample question # 5)

$$\begin{aligned} i &\approx \frac{60 + 5 + 25 + 80 + 35 - 75 - 12(10)}{\left\{ 75(1) + 10\left(\frac{11}{12} + \frac{10}{12} + \dots + \frac{1}{12} + \frac{0}{12}\right) \right.} \\ &\quad \left. - 5\left(\frac{10}{12}\right) - 25\left(\frac{6}{12}\right) - 80\left(\frac{2.5}{12}\right) - 35\left(\frac{2}{12}\right) \right\} \\ &= \frac{10}{90.8\bar{3}} = \underline{\underline{0.110092}} \end{aligned}$$