UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Actuarial Science Program
DEPARTMENT OF MATHEMATICS

Math 370 (Z) Prof. Rick Gorvett
Exam 2/FM Preparation

Loans and Sinking Funds
Review Problems

Topic A: Outstanding Loan Balance

Prospective: $B_t^p = Ra_{\overline{n-t|}}$
Retrospective: $B_t^r = R\left[a_{\overline{n|}}(1+i)^t - s_{\overline{t|}}\right]$

(1) A loan is being repaid with 25 annual payments of 300 each. With the 10th payment, the borrower pays an extra 1000, and then repays the balance over 10 years with a revised annual payment. The effective rate of interest is 8%. Calculate the amount of the revised annual payment.

(A) 157
(B) 183
(C) 234
(D) 257
(E) 383

$B_{10}^- = 300 \cdot a_{\overline{15|}.08} = 2567.8436$
$B_{10}^+ = B_{10}^- - 1{,}000 = 1567.8436 = R \cdot a_{\overline{10|}.08}$
$\Rightarrow R = \underline{233.655}$

(May 2005 FM Exam, Problem # 8)

(2) A 20-year loan of 1000 is repaid with payments at the end of each year. Each of the first ten payments equals 150% of the amount of interest due. Each of the last ten payments is *X*. The lender charges interest at an annual effective rate of 10%. Calculate *X*.

(A) 32
(B) 57
(C) 70
(D) 97
(E) 117

FOR FIRST 10 YEARS: PAY $1.50 I_t = 1.50(i \cdot B_{t-1}) = .15 B_{t-1}$

$I_t = .10 B_{t-1}$, SO $0.05 B_{t-1}$ IS PRINCIPAL.

$\Rightarrow B_t = 0.95 \cdot B_{t-1}$.

(2005 sample question # 9)

$\Rightarrow B_{10} = 1{,}000(1-.05)^{10} = 598.737$

$= X \cdot a_{\overline{10|}.10} \Rightarrow X = \underline{97.442}$

(3) A loan is amortized over five years with monthly payments at a nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment. Calculate the outstanding loan balance immediately after the 40th payment is made.

$j = .09/12 = .0075$

(A) 6751
(B) 6889
(C) 6941
(D) 7030
(E) 7344

41^{ST} PMT. $= 1000(1-.02)^{40}$
42^{ND} PMT. $= 1000(1-.02)^{41}$
$\vdots$

(2005 sample question # 16)

$$\Rightarrow B_{40} = 1000(.98)^{40}\left(\frac{1}{1.0075}\right) + \ldots + 1000(.98)^{59}\left(\frac{1}{1.0075}\right)^{20}$$
$$= 1000(.98)^{39}\left[(.972705)^1 + \ldots + (.972705)^{20}\right]$$
$$= 1000(.98)^{39} \cdot \left[\frac{.972705-(.972705)^{21}}{1-.972705}\right]$$
$$= \underline{6889.115}$$

Topic B: Amortization

Interest paid : $I_t = R\left[1 - v^{n-t+1}\right] = iB_{t-1}$
Principal repaid : $P_t = Rv^{n-t+1} = R - I_t$

(4) Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12. Calculate the principal repaid in the first payment.

(A) 444
(B) 454
(C) 464
(D) 474
(E) 484

(1) $B_3 = 559.12 \Rightarrow R = 559.12 + .08(559.12) = 603.85$
$B_0 = 603.85 \cdot a_{\overline{4}|.08} = 2000.03$
$\Rightarrow P_1 = R - I_1 = 603.85 - .08(2000.03) = \underline{443.85}$

(2) $P_t = R \cdot v^{n-t+1}$

(2005 sample question # 46)

$P_4 = 559.12 \Rightarrow \frac{P_4}{P_1} = \frac{R\cdot v^1}{R \cdot v^4} = 1/v^3 \Rightarrow P_4 = (1.08)^3 \cdot P_1$
$\Rightarrow P_1 = \underline{443.85}$

(5) A bank customer takes out a loan of 500 with a 16% nominal interest rate convertible quarterly. The customer makes payments of 20 at the end of each quarter. Calculate the amount of principal in the fourth payment.

$j = .16/4 = .04$ PER QUARTER.

(A) 0.0
(B) 0.9
(C) 2.7
(D) 5.2
(E) There is not enough information to calculate the amount of principal.

BUT, $500(.04) = 20$, SO EACH 20 PMT. ONLY PAYS INTEREST.
$\Rightarrow I_4 = 20, P_4 = 0.$

(May 2005 FM Exam, Problem # 25)

(6) A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(i) Equal annual payments at an annual effective rate of 8.07%.
(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective rate of i.

The sum of the payments under option (i) equals the sum of the payments under option (ii). Determine i.

(A) 8.75%
(B) 9.00%
(C) 9.25%
(D) 9.50%
(E) 9.75%

$(i)\ 2000 = R \cdot a_{\overline{10}|.0807} \Rightarrow 10 \cdot R = 2990.01$

$(ii)\ 2990.01 = 10(200) + i\left[2000 + 1800 + \ldots + 200\right]$

$\Rightarrow 990.01 = i \cdot (11{,}000) \Rightarrow i = \underline{\underline{.0900}}$

(2005 sample question # 15)

(7) A discount electronics store advertises the following financing arrangement: "We don't offer you confusing interest rates. We'll just divide your total cost by 10 and you can pay us that amount each month for a year." The first payment is due on the date of sale and the remaining eleven payments at monthly intervals thereafter. Calculate the effective annual interest rate the store's customers are paying on their loans.

(A) 35.1%
(B) 41.3%
(C) 42.0%
(D) 51.2%
(E) 54.9%

$\underline{\underline{PV(1)}} = \underline{\underline{PV(2)}}$

$TC = \left(\frac{TC}{10}\right) \cdot \ddot{a}_{\overline{12}|j}$, j = MONTHLY EFF. RATE.

(NOTE: $\ddot{a}_{\overline{12}|} = 1 + a_{\overline{11}|}$)

$\Rightarrow 10 = \ddot{a}_{\overline{12}|j}$

(May 2005 FM Exam, Problem # 21)

OR $9 = a_{\overline{11}|j} \Rightarrow j = 3.503153\%$ (FROM CALCULATOR)

$\Rightarrow (1+i) = (1+j)^{12} - 1 \Rightarrow i = \underline{\underline{51.16\%}}$

(8) A loan is repaid with level annual payments based on an annual effective interest rate of 7%. The 8th payment consists of 789 of interest and 211 of principal. Calculate the amount of interest paid in the 18th payment.

$\hookrightarrow R = 1000.$

(A) 415
(B) 444
(C) 556
(D) 585
(E) 612

$P_t = R \cdot v^{n-t+1}$

$\Rightarrow P_{18} = (1.07)^{10} \cdot P_8 = 415 \Rightarrow I_{18} = 1000 - 415 = \underline{\underline{585}}$

(November 2005 FM Exam, Problem # 18)

Topic C: Sinking Funds

$$\text{Basic SF}: R_s s_{\overline{n}|j} = B_0 \quad \text{and} \quad I_t = iB_0$$
$$\frac{1}{a_{\overline{n}|i}} = \frac{1}{s_{\overline{n}|i}} + i$$

(9) Lori borrows 10,000 for 10 years at an annual effective interest rate of 9%. At the end of each year, she pays the interest on the loan and deposits the level amount necessary to repay the principal to a sinking fund earning an annual effective interest rate of 8%. The total payments made by Lori over the 10-year period is X. Calculate X.

(A) 15,803
(B) 15,853
(C) 15,903
(D) 15,953
(E) 16,003

$I_t = .09(10{,}000) = 900, \quad t = 1, 2, \ldots, 10.$

$SFD \cdot s_{\overline{10}|.08} = 10000 \Rightarrow SFD = 690.295$

$\Rightarrow 10[900 + 690.295] = \underline{15{,}902.95}$

(May 2005 FM Exam, Problem # 2)

(10) John borrows 10,000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of 1,627.45 at the end of each year. Instead, John repays the 10,000 using a sinking fund that pays an annual effective interest rate of 14%. The deposits to the sinking fund are equal to 1,627.45 minus the interest on the loan and are made at the end of each year for 10 years. Determine the balance in the sinking fund immediately after repayment of the loan.

(A) 2,130
(B) 2,180
(C) 2,230
(D) 2,300
(E) 2,370

$I_t = .10(10{,}000) = 1{,}000$

$\Rightarrow SFD = 1627.45 - 1{,}000 = 627.45$

$\Rightarrow 627.45 \cdot s_{\overline{10}|.14} = 12{,}133.19$

(2005 sample question # 4)

$\Rightarrow 12{,}133.19 - 10000 = \underline{2{,}133.19}$

(11) A 20-year loan of 20,000 may be repaid under the following two methods:

i) amortization method with equal annual payments at an annual effective rate of 6.5%
ii) sinking fund method in which the lender receives an annual effective rate of 8% and the sinking fund earns an annual effective rate of j

Both methods require a payment of X to be made at the end of each year for 20 years. Calculate j.

(A) $j \le 6.5\%$
(B) $6.5\% < j \le 8.0\%$
(C) $8.0\% < j \le 10.0\%$
(D) $10.0\% < j \le 12.0\%$
(E) $j > 12.0\%$

(i) $20{,}000 = X \cdot a_{\overline{20}|.065} \Rightarrow X = 1815.1279$

$\underbrace{(1815.1279 - .08(20{,}000))}_{SFD} \cdot s_{\overline{20}|j} = 20{,}000$

$\Rightarrow 215.1279 \cdot s_{\overline{20}|j} = 20{,}000 \Rightarrow s_{\overline{20}|j} = 92.969954$

$\Rightarrow j = \underline{14.179\%}$

(2005 sample question # 24)