

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Actuarial Science Program
DEPARTMENT OF MATHEMATICS

Math 370 (Z)
 Exam 2/FM Preparation

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Bonds
Review Problems

Topic A: Bond Pricing Formulas

Basic : $P = Fra_{\overline{n}|i} + Cv_i^n$
 Prem - Disc : $P = C + (Fr - Ci)a_{\overline{n}|i}$
 Base Amt : $P = G + (C - G)v_i^n$
 Makeham : $P = K + \frac{g}{i}(C - K)$
 $G = \frac{Fr}{i}, K = Cv_i^n, g = \frac{Fr}{C}$

- 1) Susan can buy a zero coupon bond that will pay 1000 at the end of 12 years and is currently selling for 624.60 . Instead she purchases a 6% bond with coupons payable semi-annually that will pay 1000 at the end of 10 years. If she pays X she will earn the same annual effective interest rate as the zero coupon bond. Calculate X .

- (A) 1164
 (B) 1167
 (C) 1170
 (D) 1173
 (E) 1176

$$i = \left(\frac{1000}{624.60}\right)^{1/12} - 1 = 0.0400$$

$$j = (1.04)^{1/2} - 1 = .019804$$

(May 2005 FM Exam, question # 5)

$$X = 30 \cdot a_{\overline{20}|j} + 1000 \cdot v_j^{20} = \underline{\underline{1167.04}}$$

- 2) A ten-year 100 par value bond pays 8% coupons semiannually. The bond is priced at 118.20 to yield an annual nominal rate of 6% convertible semiannually. Calculate the redemption value of the bond.

- (A) 97
 (B) 100
 (C) 103
 (D) 106
 (E) 109

$$118.20 = 4 \cdot a_{\overline{20}|.03} + C \cdot v_{.03}^{20}$$

$$\Rightarrow C = \underline{\underline{106.00}}$$

(November 2005 FM Exam, question # 4)

- 3) A 30-year bond with a par value of 1000 and 12% coupons payable quarterly is selling at 850. Calculate the annual nominal yield rate convertible quarterly.

- (A) 3.5%
 (B) 7.1%
 (C) 14.2%
 (D) 14.9%
 (E) 15.4%

$$850 = 30 \cdot a_{\overline{120}|j} + 1000 \cdot v_j^{120}$$

$$\Rightarrow j = 3.54\% \text{ (PER CALCULATOR)}$$

(November 2005 FM Exam, question # 24)

$$\Rightarrow i^{(4)} = 4j = \underline{\underline{14.16\%}}$$

- 4) An investor borrows an amount at an annual effective interest rate of 5% and will repay all interest and principal in a lump sum at the end of 10 years. She uses the amount borrowed to purchase a 1000 par value 10-year bond with 8% semiannual coupons bought to yield 6% convertible semiannually. All coupon payments are reinvested at a nominal rate of 4% convertible semiannually. Calculate the net gain to the investor at the end of 10 years after the loan is repaid.

- (A) 96
 (B) 101
 (C) 106
 (D) 111
 (E) 116

$$\text{AMT. BORROWED} = PV = 40 \cdot a_{\overline{20}|.03} + 1000 \cdot v_{.03}^{20}$$

$$= 1,148.775 = B_0$$

$$\text{@ } t = 20: \text{ PAYOFF OF LOAN: } -B_0 (1.05)^{10} = -1871.23$$

(November 2005 FM Exam, question # 11)

$$\text{BOND: } 1000 + 40 \cdot s_{\overline{20}|.02} = \frac{1971.89}{100.66}$$

- 5) Dan purchases a 1000 par value 10-year bond with 9% semiannual coupons for 925. He is able to reinvest his coupon payments at a nominal rate of 7% convertible semiannually. Calculate his nominal annual yield rate convertible semiannually over the ten-year period.

- (A) 7.6%
 (B) 8.1%
 (C) 9.2%
 (D) 9.4%
 (E) 10.2%

$$A(20) = 1000 + 45 \cdot s_{\overline{20}|.035} = 2,272.586$$

$$\left(\frac{2,272.586}{925} \right)^{1/20} - 1 = j = .045969$$

(November 2005 FM Exam, question # 16)

$$i^{(2)} = 2j = \underline{\underline{.0919}}$$

- 6) You have decided to invest in Bond X, an n -year bond with semi-annual coupons and the following characteristics:

- Par value is 1000.
- The ratio of the semi-annual coupon rate to the desired semi-annual yield rate, r/i , is 1.03125. $= r/i$
- The present value of the redemption value is 381.50. $= C \cdot v^{2n}$

Given $v^n = 0.5889$, what is the price of bond X?

- (A) 1019
 (B) 1029
 (C) 1050
 (D) 1055
 (E) 1072

BASE AMT. FORMULA: $P = G + (C - G)v_i^{2n}$, $G = Fr/i$
 $C = 381.50 / (.5889)^2 = 1,100.05 = 1031.25$
 $\Rightarrow P = \underline{\underline{1055.11}}$

(2005 sample question # 22)

OR: $P = 1000 \cdot r \cdot a_{\overline{2n}|i} + C v_i^{2n}$
 $= 1000 \left(\frac{r}{i} \right) (1 - v_i^{2n}) + 381.50 = \underline{\underline{1055.11}}$

- 7) Bill buys a 10-year 1000 par value 6% bond with semi-annual coupons. The price assumes a nominal yield of 6%, compounded semi-annually. As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of i . At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond. Calculate i .

- (A) 9.50%
 (B) 9.75%
 (C) 10.00%
 (D) 10.25%
 (E) 10.50%

$P = 1000$ (since 3% coupon = 3% $\frac{1}{2}$ -yr. EFF. RATE).
 LET $j = (1+i)^{\frac{1}{2}} - 1$.
 $A(20) = 1000 + 30 \cdot s_{\overline{20}|j} = 1000(1.07)^{10}$.
 (2005 sample question # 47)
 $\Rightarrow s_{\overline{20}|j} = 32.238379 \Rightarrow j = 4.759658$
 $\Rightarrow i = \underline{\underline{9.7459\%}}$

Topic B: Premium/Discount

- 8) A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%. Calculate the interest portion of the 7th coupon. (AT A PREMIUM)

- (A) 632
 (B) 642
 (C) 651
 (D) 660
 (E) 667

BOOK VALUE: $BV_6 = 800 \cdot a_{\overline{7}|.06} + 10,000 \cdot v_{.06}^4 = 10,693.02$
 $I_7 = i \cdot BV_6 = .06(10,693.02) = \underline{\underline{641.58}}$
 (NOTE: $(800 - 641.58) = 158.42 \rightarrow BV_6 - BV_7 = 158.42$ (AMORT. OF PREMIUM)).
 (2005 sample question # 10)

NOTE: BOND BV = PV OF REMAINING CASH FLOWS AT ORIGINAL BOND YIELD.

Topic C: Callable Bonds

- 9) Matt purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of 8% convertible semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The price guarantees that Matt will receive a nominal annual rate of interest convertible semiannually of at least 6%. Calculate X. (CALLED @ END OF YEAR 15)

- (A) 1400
 (B) 1420
 (C) 1440
 (D) 1460
 (E) 1480

$1722.25 = .04 \cdot X \cdot a_{\overline{30}|.03} + X \cdot v_{.03}^{30} = X [.724018 + .411987]$
 $\Rightarrow X = \underline{\underline{1440.00}}$
 (2005 sample question # 54)

[NOTE: CALLED @ END OF YEAR 20:
 $1722.25 = 57.6 \cdot a_{\overline{40}|j} + 1440 v_j^{40}$
 $\Rightarrow j = 3.134\% \Rightarrow$ WHEN AT PREMIUM, EARLIEST CALL DATE IS CRITICAL.]

10) Toby purchased a 20-year par value bond with semiannual coupons at a nominal annual rate of 8% convertible semiannually at a price of 1722.25. The bond can be called at par value 1100 on any coupon date starting at the end of year 15. What is the minimum yield that Toby could receive, expressed as a nominal annual rate of interest convertible semiannually?

- (A) 3.2%
- (B) 3.3%
- (C) 3.4%
- (D) 3.5%
- (E) 3.6%

AT PREMIUM → WILL CALL AT EARLIEST POSSIBLE DATE
(SINCE CALL PRICE = PAR AND REDEMPTION VALUE).

$$1722.25 = 44 \cdot a_{\overline{30}|j} + 1100v_j^{30} \Rightarrow j = 1.6082\%$$

(2005 sample question # 55)

$$i^{(2)} = 2j = \underline{\underline{3.2165\%}}$$

11) A 1000 par value bond with coupons at 9% payable semiannually was called for 1100 prior to maturity. The bond was bought for 918 immediately after a coupon payment and was held to call. The nominal yield rate convertible semiannually was 10%. Calculate the number of years the bond was held.

- (A) 10
- (B) 25
- (C) 39
- (D) 49
- (E) 54

$$918 = 45 \cdot a_{\overline{n}|0.05} + 1100 \cdot v_{0.05}^n$$

$$\Rightarrow n = 50 \text{ HALF-YEARS.}$$

$$\Rightarrow \underline{\underline{25 \text{ YEARS.}}}$$

(November 2005 FM Exam, question # 22)

12) A 1000 par value bond pays annual coupons of 80. The bond is redeemable at par in 30 years, but is callable any time from the end of the 10th year at 1050. Based on her desired yield rate, an investor calculates the following potential purchase prices, P:

Assuming the bond is called at the end of the 10th year, $P = 957$

Assuming the bond is held until maturity, $P = 897$

→ 8.9991% yield (10 yrs.)
→ 9.0028% yield (held 30 yrs.)
→ ONLY 8.4% if $P = 957$ BUT HELD 30 YRS.

The investor buys the bond at the highest price that guarantees she will receive at least her desired yield rate regardless of when the bond is called. The investor holds the bond for 20 years, after which time the bond is called. Calculate the annual yield rate the investor earns.

- (A) 8.56%
- (B) 9.00%
- (C) 9.24%
- (D) 9.53%
- (E) 9.99%

$$n = 20, PV = 897, PMT = (-) 80, FV = (-) 1050$$

$$\Rightarrow j = \underline{\underline{.09243}}$$

(May 2005 FM Exam, question # 11)