

**UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN**  
**Actuarial Science Program**  
**DEPARTMENT OF MATHEMATICS**

Math 370 (Z)  
 Exam 2/FM Preparation

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**Stock Pricing, Short Sales, and Yield Curves**  
Review Problems

Topic A: Stock Pricing

Gordon Growth Model :  $P_0 = \frac{DIV_1}{i - g}$

- 1) The stock of Company X sells for 75 per share assuming an annual effective interest rate of  $i$ . Annual dividends will be paid at the end of each year forever. The first dividend is 6, with each subsequent dividend 3% greater than the previous year's dividend. Calculate  $i$ .

- (A) 8%
- (B) 9%
- (C) 10%
- (D) 11%
- (E) 12%

$$75 = \frac{6}{i - .03} \Rightarrow i = \underline{\underline{0.11}}$$

*(May 2005 FM Exam, question # 23)*

- 2) You are considering the purchase of a share of stock. If you buy the share, you will expect to receive the following dividends: \$2 one year from now, \$2.50 two years from now, \$3 three years from now, and thereafter annual dividends increase by 5% per year. If the effective annual interest rate is 11%, what is the current value of this share of stock? *(From a prior Math 210 class)*

$$P_0 = \frac{2}{1.11} + \frac{2.50}{(1.11)^2} + \left( \frac{3}{.11 - .05} \right) \cdot v_{.11}^2 = \underline{\underline{44.41}}$$

- 3) The dividends of a common stock are expected to be 1 at the end of each of the next 5 years and 2 for each of the following 5 years. The dividends are expected to grow at a fixed rate of 2% per year thereafter. Assume an annual effective interest rate of 6%. Calculate the price of this stock using the dividend discount model.

- (A) 29
- (B) 33
- (C) 37
- (D) 39
- (E) 41

$$P_0 = 1 \cdot a_{\overline{5}|.06} + 2 \cdot a_{\overline{5}|.06} \cdot v_{.06}^5 + \left( \frac{2(1.02)}{.06 - .02} \right) \cdot v_{.06}^{10} = \underline{\underline{38.99}}$$

*(November 2005 FM Exam, question # 20)*

**Topic B: Short Sales**

General approach: $r = \frac{(P_0 - P_1) + iM - DIV}{M}$ $M = \text{margin} = P_0 \times (\text{margin}\%)$
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- 4) Eric and Jason each sell a different stock short at the beginning of the year for a price of 800. The margin requirement for each investor is 50% and each will earn an annual effective interest rate of 8% on his margin account. Each stock pays a dividend of 16 at the end of the year. Immediately thereafter, Eric buys back his stock at a price of  $(800 - 2X)$ , and Jason buys back his stock at a price of  $(800 + X)$ . Eric's annual effective yield,  $i$ , on the short sale is twice Jason's annual effective yield. Calculate  $i$ .

- (A) 4%
- (B) 6%
- (C) 8%
- (D) 10%
- (E) 12%

$MARGIN = .50(800) = 400$   
 $INTEREST = 400(.08) = 32$   
 ERIC:  $i = \frac{800 - (800 - 2X) + 32 - 16}{400} = \frac{X + 8}{200}$   
 JASON:  $\frac{1}{2}i = \frac{800 - (800 + X) + 32 - 16}{400} = \frac{16 - X}{400}$   
 (2005 sample question # 38) }  $x = 4$   
}  $i = \underline{\underline{.06}}$

- 5) Jose and Chris each sell a different stock short for the same price. For each investor, the margin requirement is 50% and interest on the margin debt is paid at an annual effective rate of 6%. Each investor buys back his stock one year later at a price of 760. Jose's stock paid a dividend of 32 at the end of the year while Chris's stock paid no dividends. During the 1-year period, Chris's return on the short sale is  $i$ , which is twice the return earned by Jose. Calculate  $i$ .

- (A) 12%
- (B) 16%
- (C) 18%
- (D) 20%
- (E) 24%

$MARGIN = .50P$      $INTEREST = .50P(.06)$   
 CHRIS:  $i = \frac{P - 760 + .03P}{.50P} = \frac{1.03P - 760}{.50P}$   
 JOSE:  $\frac{1}{2}i = \frac{P - 760 + .03P - 32}{.50P} = \frac{1.03P - 792}{.50P}$   
 (2005 sample question # 39)  
 $\Rightarrow P = 800, i = \underline{\underline{0.16}}$

- 6) Bill and Jane each sell a different stock short for a price of 1000. For both investors, the margin requirement is 50%, and interest on the margin is credited at an annual effective rate of 6%. Bill buys back his stock one year later at a price of  $P$ . At the end of the year, the stock paid a dividend of  $X$ . Jane also buys back her stock after one year, at a price of  $(P - 25)$ . At the end of the year, her stock paid a dividend of  $2X$ . Both investors earned an annual effective yield of 21% on their short sales. Calculate  $P$ .

- (A) 800
- (B) 825
- (C) 850
- (D) 875
- (E) 900

$MARGIN = .5(1000) = 500$      $INTEREST = .06(500) = 30$   
 BILL:  $0.21 = \frac{1000 - P + 30 - X}{500} \Rightarrow X = 925 - P$   
 JANE:  $0.21 = \frac{1000 - (P - 25) + 30 - 2X}{500} \Rightarrow X = \frac{950 - P}{2}$   
 (2005 sample question # 40)  
 $\Rightarrow P = \underline{\underline{900}}$

- 7) On January 1, 2004, Karen sold stock A short for 50 with a margin requirement of 80%. On December 31, 2004, the stock paid a dividend of 2, and an interest amount of 4 was credited to the margin account. On January 1, 2005, Karen covered the short sale at a price of  $X$ , earning a 20% return. Calculate  $X$ .

- (A) 40  
 (B) 44  
 (C) 48  
 (D) 52  
 (E) 56

$$.20 = \frac{50 - X + 4 - 2}{40} \Rightarrow X = \underline{\underline{44}}$$

(May 2005 FM Exam, question # 22)

- 8) Theo sells a stock short with a current price of 25,000 and buys it back for  $X$  at the end of 1 year. Governmental regulations require the short seller to deposit margin of 40% at the time of the short sale. The prevailing interest rate is an 8% annual rate, and Theo earns a 25% yield on the transaction. Calculate  $X$ .

- (A) 19,550  
 (B) 20,750  
 (C) 22,500  
 (D) 23,300  
 (E) 24,500

$$.25 = \frac{25,000 - X + .08(.4(25,000))}{.4(25,000)} \Rightarrow X = \underline{\underline{23,300}}$$

(November 2005 FM Exam, question # 17)

**Topic C: Yield Curves**

- 9) You are given the following information:

Term	Annual Spot Interest Rates
1	7%
2	8%
3	9%

Given this term structure, what is the one-year forward rate beginning one year from now?

$$(1.08)^2 = (1.07)(1 + f_1) \Rightarrow f_1 = \underline{\underline{.0901}}$$

- 10) Given the term structure in the prior problem, what is the one-year forward rate beginning two years from now?

$$(1.09)^3 = (1.08)^2(1 + f_2) \Rightarrow f_2 = \underline{\underline{.1103}}$$

11) Yield rates to maturity for zero coupon bonds are currently quoted at 8.5% for one-year maturity, 9.5% for two-year maturity, and 10.5% for three-year maturity. Let  $i$  be the one-year forward rate for year two implied by current yields of these bonds. Calculate  $i$ .

- (A) 8.5%
- (B) 9.5%
- (C) 10.5%
- (D) 11.5%
- (E) 12.5%

$$(1.095)^2 = (1.085)(1+i) \Rightarrow i = \underline{\underline{.10509}}$$

(May 2005 FM Exam, Problem # 10)

12) Consider a yield curve defined by the following equation:

$$i_k = 0.09 + 0.002k - 0.001k^2$$

where  $i_k$  is the annual effective rate of return for zero coupon bonds with maturity of  $k$  years. Let  $j$  be the one-year effective rate during year 5 that is implied by this yield curve. Calculate  $j$ .

- (A) 4.7%
- (B) 5.8%
- (C) 6.6%
- (D) 7.5%
- (E) 8.2%

$$(1+i_4)^4(1+j) = (1+i_5)^5$$

$$i_4 = .082 \quad i_5 = .075 \quad \Rightarrow j = \underline{\underline{4.745\%}}$$

(November 2005 FM Exam, Problem # 6)

13) You are given the following term structure of spot interest rates:

Term (in years)	Spot interest rate
1	5.00%
2	5.75%
3	6.25%
4	6.50%

A three-year annuity-immediate will be issued a year from now with annual payments of 5000. Using the forward rates, calculate the present value of this annuity a year from now.

- (A) 13,094
- (B) 13,153
- (C) 13,296
- (D) 13,321
- (E) 13,401

$$PV_{t=1} = 5000 \left[ \frac{1}{(1.0575)^2 / 1.05} + \frac{1}{(1.0625)^3 / 1.05} + \frac{1}{(1.065)^4 / 1.05} \right]$$

(November 2005 FM Exam, Problem # 15)

$$= \underline{\underline{13,153}}$$