UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Actuarial Science Program
DEPARTMENT OF MATHEMATICS

Math 370 (Z) Prof. Rick Gorvett
Exam 2/FM Preparation

Inflation, Duration, and Immunization
Review Problems

Topic A: Inflation

$(1+i) = (1+i')(1+r)$
where:
i = nominal (market) rate of interest
i' = real (after inflation) rate of interest
r = rate of inflation

$$PV = R(1+r)\left[\frac{1-\left(\frac{1+r}{1+i}\right)^n}{i-r}\right] = Ra_{\overline{n}|i'}$$

where:
PV = present value at t = 0
$R(1+r)$ = first payment, at t = 1
n = number of payments, increasing geometrically by (1 + r)

1) An insurance company has an obligation to pay the medical costs for a claimant. Average annual claims costs today are $5,000, and medical inflation is expected to be 7% per year. The claimant is expected to live an additional 20 years. Claim payments are made at yearly intervals, with the first claim payment to be made one year from today. Find the present value of the obligation if the annual interest rate is 5%.

(A) 87,932
(B) 102,514
(C) 114,611
(D) 122,634
(E) Cannot be determined

(2005 sample question # 31)

$PV = 5000\left[1.07v^1 + \ldots + (1.07)^{20}v^{20}\right]$

LET $x = 1.07/1.05 \Rightarrow PV = 5000\left[\frac{x - x^{21}}{1-x}\right]$

$= \underline{122{,}634}$

OR: $i = .05,\ r = .07 \Rightarrow i' = \left(\frac{1+i}{1+r}\right) - 1 = -.018692$

$\Rightarrow 5000 \cdot a_{\overline{20}|i'} = \underline{122{,}634}$

Topic B: Duration

$$\bar{d} = \frac{\sum_{t=1}^{n} t v^t CF_t}{\sum_{t=1}^{n} v^t CF_t}$$

$$\bar{v} = \frac{-P'(i)}{P(i)} = \frac{\bar{d}}{1+i}$$

where:

$\bar{d}$ = (Macaulay) duration

$\bar{v}$ = volatility (or Modified duration)

2) The current price of an annual coupon bond is 100. The derivative of the price of the bond with respect to the yield to maturity is -700. The yield to maturity is an annual effective rate of 8%. Calculate the duration of the bond.

(A) 7.00
(B) 7.49
(C) 7.56
(D) 7.69
(E) 8.00

$P'(i) = -700$

$P(i) = 100 \qquad i = .08$

$\bar{v} = -\frac{P'(i)}{P(i)} = \frac{-(-700)}{100} = 7 = \bar{d}/(1+i) \Rightarrow \bar{d} = \underline{7.56}$

(2005 sample question # 35)

3) Calculate the duration of a common stock that pays dividends at the end of each year into perpetuity. Assume that the dividend is constant, and that the effective rate of interest is 10%.

(A) 7
(B) 9
(C) 11
(D) 19
(E) 27

$\bar{d} = \frac{D \cdot v^1(1) + D \cdot v^2(2) + \ldots}{D \cdot v^1 + D \cdot v^2 + \ldots} = \frac{v^1 + 2 \cdot v^2 + \ldots}{v^1 + v^2 + \ldots}$

$= \frac{(Ia)_{\overline{\infty}|.10}}{a_{\overline{\infty}|.10}} = \left(\frac{1}{i} + \frac{1}{i^2}\right) / \left(\frac{1}{i}\right) = 1 + \frac{1}{i} = \underline{11}$

(2005 sample question # 36)

4) Calculate the duration of a common stock that pays dividends at the end of each year into perpetuity. Assume that the dividend increases by 2% each year and that the effective rate of interest is 5%.

(A) 27
(B) 35
(C) 44
(D) 52
(E) 58

$\bar{v} = \frac{-P'(i)}{P(i)} \qquad P(i) = \frac{D}{i-g} \Rightarrow P'(i) = \frac{-D}{(i-g)^2}$

(2005 sample question # 37)

$\Rightarrow \bar{v} = \frac{1}{i-g} = \frac{\bar{d}}{1+i}$

$\Rightarrow \bar{d} = \bar{v}(1+i) = \left(\frac{1}{.05-.02}\right)(1.05)$

$\left(\text{or } \frac{1+i}{i-g}\right) = \underline{35}$

5) A bond will pay a coupon of 100 at the end of each of the next three years and will pay the face value of 1000 at the end of the three-year period. The bond's duration (Macaulay duration) when valued using an annual effective interest rate of 20% is *X*. Calculate *X*.

(A) 2.61
(B) 2.70
(C) 2.77
(D) 2.89
(E) 3.00

$$\bar{d} = \frac{\sum t \cdot v^t \cdot C_t}{\sum v^t \cdot C_t} = \frac{100(1)v^1 + 100(2)v^2 + 1100(3)v^3}{100v^1 + 100v^2 + 1100v^3}$$

$$= \underline{\underline{2.701}}$$

(May 2005 FM Exam, Problem # 3)

6) Calculate the Macaulay duration of an eight-year 100 par value bond with 10% annual coupons and an effective rate of interest equal to 8%.

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

$$\bar{d} = \frac{10 \cdot (Ia)_{\overline{8}|.08} + 800 \cdot v^8}{10 \cdot a_{\overline{8}|.08} + 100v^8} = \frac{667.742}{111.493} = \underline{\underline{5.99}}$$

(November 2005 FM Exam, Problem # 2)

7) John purchased three bonds to form a portfolio as follows:

Bond A has semi-annual coupons at 4%, a duration of 21.46 years, and was purchased for 980.
Bond B is a 15-year bond with a duration of 12.35 years and was purchased for 1015.
Bond C has a duration of 16.67 years and was purchased for 1000.

Calculate the duration of the portfolio at the time of purchase.

(A) 16.62 years
(B) 16.67 years
(C) 16.72 years
(D) 16.77 years
(E) 16.82 years

$$\frac{21.46(980) + 12.35(1015) + 16.67(1000)}{980 + 1015 + 1000}$$

(May 2005 FM Exam, Problem # 6)

$$= \underline{\underline{16.773}}$$

Topic C: Immunization

The following information applies to questions ~~1~~ 8 thru ~~3~~ 10: $\rightarrow L_{1/2} = 1000$

Joe must pay liabilities of 1,000 due 6 months from now and another 1,000 due one year from now. There are two available investments: $\rightarrow L_1 = 1000$

- I: a 6-month bond with face amount of 1,000, a 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually; and $\} A_{1/2} = 1040$
- II: a one-year bond with face amount of 1,000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually $\} A_{1/2} = 25$, $A_1 = 1025$

8) How much of each bond should Joe purchase in order to exactly (absolutely) match the liabilities?

	Bond I	Bond II
(A)	1	.97561
(B)	.93809	1
(C)	.97561	.94293
(D)	.93809	.97561
(E)	.98345	.97561

	0	1/2	1
NEED:		1000	1000
BONDS: I:		1040	
II:		25	1025

(2005 sample question # 51)

$@\ t=1$, NEED $\frac{1000}{1025} = .975610$ of II.

$@\ t=\frac{1}{2}$, NEED x of I: $x(1040) + .975610(25) = 1000$ $\Rightarrow x = \underline{.938086}$

9) What is Joe's total cost of purchasing the bonds required to exactly (absolutely) match the liabilities?

(A) 1894
(B) 1904
(C) 1914
(D) 1924
(E) 1934

$P_I = \frac{1040}{1.03} = 1009.71$

$P_{II} = \frac{25}{1.035} + \frac{1025}{(1.035)^2} = 981.00$

(2005 sample question # 52)

$\Rightarrow .938086(1009.71) + .975610(981.00) = \underline{1904.26}$

10) What is the annual effective yield rate for investment in the bonds required to exactly (absolutely) match the liabilities?

(A) 6.5%
(B) 6.6%
(C) 6.7%
(D) 6.8%
(E) 6.9%

$1904.26 = 1000 \cdot v_j + 1000 \cdot v_j^2 \Rightarrow v_j = .967740$

$\Rightarrow j = 1.0333$ / HALF-YR.

(2005 sample question # 53)

$\Rightarrow 1+i = (1+j)^2 \Rightarrow i = \underline{6.778\%}$

(OR: $1904.26 = 1000 \cdot a_{\overline{2}|j}$)

11) An insurance company accepts an obligation to pay 10,000 at the end of each year for 2 years. The insurance company purchases a combination of the following two bonds at a total cost of X in order to exactly match its obligation:

(i) 1-year 4% annual coupon bond with a yield rate of 5% (F_1)
(ii) 2-year 6% annual coupon bond with a yield rate of 5%. (F_2)

0	1	2
	10,000	10,000
	$1.04 F_1$	
	$.06 F_2$	$1.06 F_2$

Calculate X.

$1.06 F_2 = 10{,}000 \Rightarrow F_2 = \underline{9433.9624}$

$\Rightarrow 1.04 F_1 + .06 F_2 = 10{,}000$

$\Rightarrow F_1 = 9071.1176$

(A) 18,564
(B) 18,574
(C) 18,584
(D) 18,594
(E) 18,604

(May 2005 FM Exam, Problem # 15)

$$\frac{1.04F_1}{1.05} + \frac{.06F_2}{1.05} + \frac{1.06F_2}{(1.05)^2} = \underline{\underline{18{,}594.10}}$$

EASIER: SINCE BOTH BOND YIELDS ARE 5%.

$\frac{10000}{1.05} + \frac{10{,}000}{(1.05)^2} = 18{,}594.10$

12) A company must pay liabilities of 1000 and 2000 at the end of years 1 and 2, respectively. The only investments available to the company are the following two zero-coupon bonds:

Maturity (years)	Effective annual yield	Par	Price
1	10%	1000	909.09
2	12%	1000	797.19

Determine the cost to the company today to match its liabilities exactly.

(A) 2007
(B) 2259
(C) 2503
(D) 2756
(E) 3001

$\text{PRICE} = (1)(909.09) + (2)(797.19) = \underline{\underline{2503.47}}$

(November 2005 FM Exam, Problem # 10)

13) Which of the following statements about zero-coupon bonds are true?

I. Zero-coupon bonds may be created by separating the coupon payments and redemption values from bonds and selling each of them separately.
II. The yield rates on stripped Treasuries at any point in time provide an immediate reading of the risk-free yield curve.
III. The interest rates on the risk-free yield curve are called ~~forward~~ Spot rates.

(A) I only
(B) II only
(C) III only
(D) I, II, and III
(E) The correct answer is not given by (A), (B), (C), or (D).

(November 2005 FM Exam, Problem # 19)

14) Which of the following statements about immunization strategies are true?

I. To achieve immunization, the convexity of the assets must (>) ~~equal~~ the convexity of the liabilities.

II. The full immunization technique is designed to work for any change in the interest rate.

III. The theory of immunization was developed to protect against adverse effects created by changes in interest rates.

(A) None
(B) I and II only
(C) I and III only
(D) II and III only
(E) The correct answer is not given by (A), (B), (C), and (D).

(November 2005 FM Exam, Problem # 21)