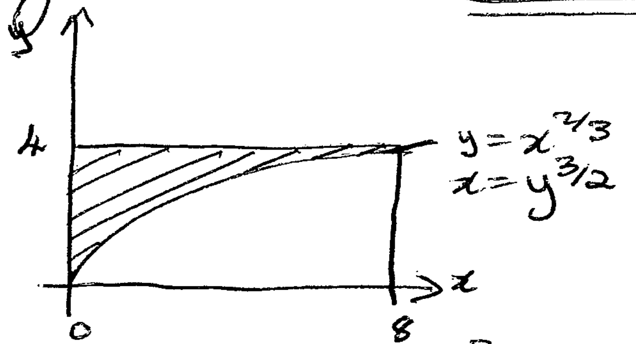
Spring 2007, Merit Math 241, Practice Exam 3

1.

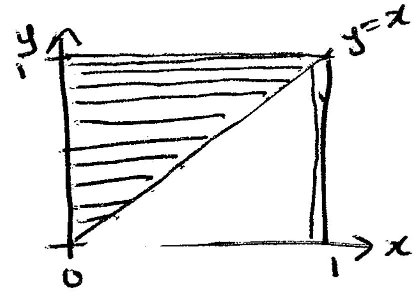


$$\int_0^8 \int_{x^{2/3}}^4 x \cos y^4 \, dy \, dx = \int_0^4 \int_0^{y^{3/2}} x \cos y^4 \, dx \, dy$$

$$= \int_0^4 \left[\tfrac{1}{2} x^2 \cos y^4\right]_0^{y^{3/2}} dy = \int_0^4 \tfrac{1}{2} y^3 \cos y^4 \, dy$$

$$= \tfrac{1}{8} \sin y^4 \Big|_0^4 = \underline{\frac{\sin(4^4)}{8}}.$$

2.



$$\text{Surface Area} = \iint_R \sqrt{1 + (z_x)^2 + (z_y)^2} \, dx \, dy$$

$$\int_0^1 \int_0^y \sqrt{1 + 1 + 4y^2} \, dx \, dy$$

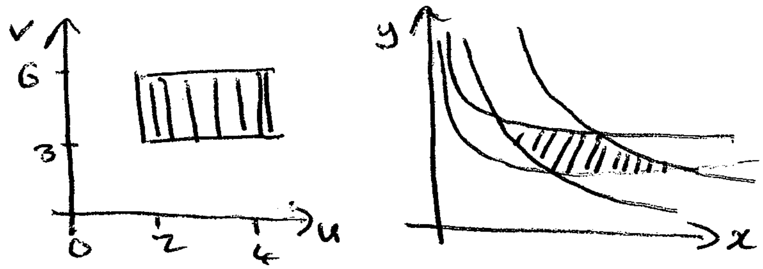
$$= \int_0^1 y\sqrt{2+4y^2} \, dy = \underline{\tfrac{1}{12}(2+4y^2)^{3/2}} = \tfrac{1}{12}(18)^{3/2} - \tfrac{1}{12}(2)^{3/2}.$$

3.

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} y & x \\ y^3 & 3xy^2 \end{vmatrix}} = \frac{1}{3xy^3 - xy^3}$$

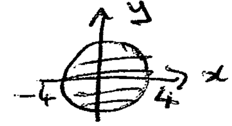
$$= \frac{1}{2xy^3} = \frac{1}{2v}.$$



$$\int_3^6 \int_2^4 \frac{1}{2v} \, du \, dv = \int_3^6 \frac{1}{v} \, dv = \log 6 - \log 3 \quad (= \log 2).$$

4. The paraboloids intersect when

$$2x^2+2y^2 = 48 - x^2 - y^2 \Rightarrow x^2+y^2 = 4^2.$$

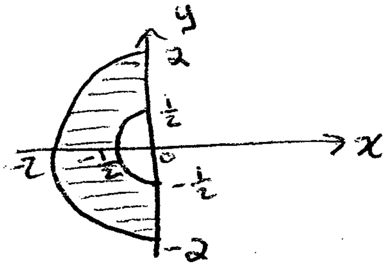


Using cylindrical coordinates,

$$\int_0^{2\pi}\int_0^4\int_{2r^2}^{48-r^2} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^4 48r - 3r^3\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[24r^2 - \tfrac{3}{4}r^4\right]_0^4 d\theta = \int_0^{2\pi} 384 - 192\,d\theta = 384\pi.$$

5.



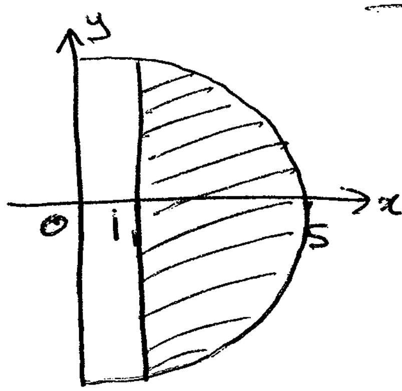
One way to do this is:

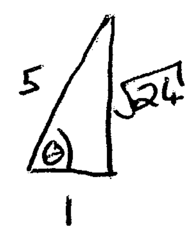
$$\int_{-2}^{-\frac{1}{2}}\int_{-\sqrt{4-y^2}}^{0}\frac{1}{(x^2+y^2)^2}\,dx\,dy$$

$$+\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\sqrt{\frac{1}{4}-y^2}}^{-\sqrt{\frac{1}{4}-y^2}}\frac{1}{(x^2+y^2)^2}\,dx\,dy$$

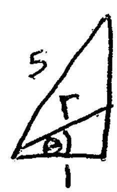
$$+\int_{\frac{1}{2}}^{2}\int_{-\sqrt{4-y^2}}^{0}\frac{1}{(x^2+y^2)^2}\,dx\,dy.$$

6.





$\theta = \cos^{-1}\frac{1}{5}$

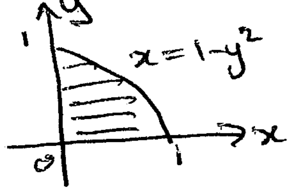


$\cos\theta = \frac{1}{r}$

$r = \frac{1}{\cos\theta}$

$$\int_{-\cos^{-1}\frac{1}{5}}^{\cos^{-1}\frac{1}{5}}\int_{\frac{1}{\cos\theta}}^{5} r^4\cos\theta\sin^2\theta\,dr\,d\theta.$$

7.



(a) mass $= \iint_R$ density dA

$$\int_0^1 \int_0^{1-y^2} y \, dx\, dy = m$$

(b) $\bar{x} = \frac{1}{m} \iint_R x \, \delta(x,y)\, dA$

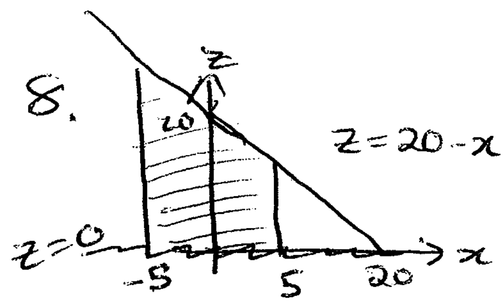
$$\bar{x} = \frac{1}{m} \int_0^1 \int_0^{1-y^2} xy \, dx\, dy$$

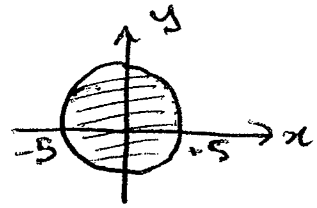
$$\bar{y} = \frac{1}{m} \int_0^1 \int_0^{1-y^2} y^2 \, dx\, dy$$

(c) $I_0 = \iint_R r^2 \delta \, dA = \iint_R (x^2+y^2)\delta \, dA$

$$= \int_0^1 \int_0^{1-y^2} (x^2+y^2) y \, dx\, dy$$

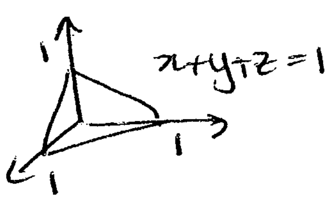
8.





(a) In Cylindrical: $\int_0^{2\pi} \int_0^5 \int_0^{20-r\cos\theta} r \, dz\, dr\, d\theta$.

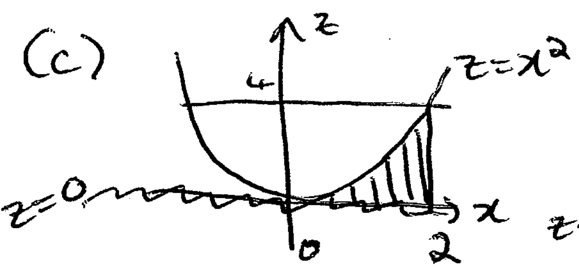
(b)

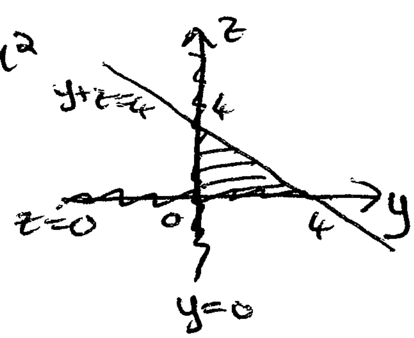


$$m = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} (xy+z^2)\, dz\, dy\, dx$$

$$\begin{matrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{matrix} = \frac{1}{m} \int_0^1 \int_0^{1-x} \int_0^{1-x-y} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} (xy+z^2)\, dz\, dy\, dx$$

(c)





$$\int_0^2 \int_0^{x^2} \int_0^{4-z} 1 \, dy\, dz\, dx.$$