Math 241, Spring 2007, Merit Worksheet 24

1. What does it mean for a line integral to be independent of path? Evaluate *one* of the following:

   (a) $\int_C xy\ dx + xz\ dz$ where C is the curve $\mathbf{r}(t) =< t^3e^t, \log(t^2+1), \sqrt{t} >$ as t goes from 0 to 1.

   (b) $\int_C (3x^2y^2+z^2)dx + (2x^3y+z^4)dy + (4yz^3+2xz)dz$ where C is the same curve as above.

2. Show the line integral below is independent of path and then calculate the value of the line integral:

$$\int_{(0,0)}^{(1,-1)} (e^y + ye^x)dx + (e^x + xe^y)dy.$$

3. Suppose that the vector field $\mathbf{F} = \nabla f$ for some scalar function f. Suppose C is the curve $\mathbf{r}(t) =< \cos t, \sin t >$ for $0 \leq t \leq 2\pi$. Let C_1 be that portion of the curve for $0 \leq t \leq \frac{3\pi}{2}$ and let C_2 be that portion of the curve given by $\frac{3\pi}{2} \leq t \leq 2\pi$. Find $\int_C \mathbf{F} \cdot \mathbf{T}\ ds$. What's the relationship between $\int_{C_1} \mathbf{F} \cdot \mathbf{T}\ ds$ and $\int_{C_2} \mathbf{F} \cdot \mathbf{T}\ ds$?

4. Use Green's Theorem to evaluate the line integral

$$\int_C (1 + y + e^{-x^2})\ dx\ +\ (2x + \cos y^5)\ dy,$$

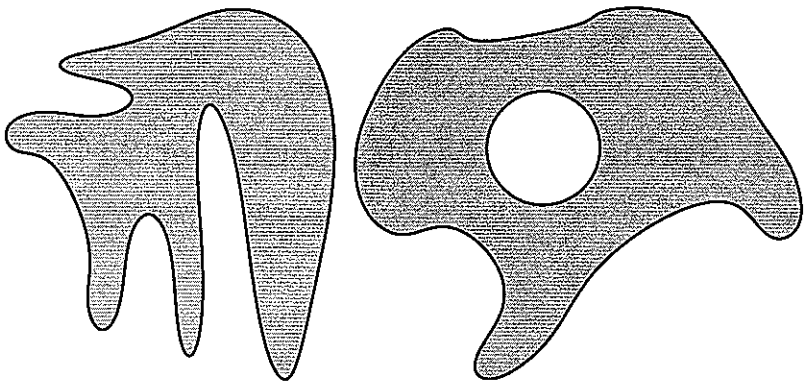
   where C is the curve obtained by going around the square $\{(x,y)$: $-1 \leq x \leq 1, -1 \leq y \leq 1$ in a counterclockwise direction.

5. Use (a corollary of) Green's Theorem to find the area of the region

   (a) the circle bounded by $x = a\cos t, y = a\sin t,\ 0 \leq t \leq 2\pi$.

   (b) the region between the graphs of $y = x^3$ and $y = x^4$.

6. Use the pictures below to indicate the direction of the positively oriented boundary curve of the regions. Also indicate the direction of the outer normal vector.



7. Evaluate (using Green's Theorem)

$$\int_C xy\ dx + x^2\ dy,$$

where C is curve going *clockwise* around the first-quadrant loop of the graph of the polar equation $r = \sin 2\theta$.

Warm-up for next time

Write out all the versions of Green's Theorem.

Next week's office hours will be held in my office (150 Altgeld) rather than in the Merit Room (173 Altgeld)