

Math 241, Spring 2007, Merit Worksheet 22

1. Illustrate the following vector fields:

(a) $2\vec{i} + x\vec{j}$

(b) $x\vec{i} - y\vec{j}$

(c) $(x^2 + y^2)^{-1/2}(x\vec{i} + y\vec{j})$

(d) $y\vec{i} - x\vec{j}$

2. Find the gradient vector field ∇f if f is given by $f(x, y, z) = xy + yz^2 + y + 3$. Find and plot the vector field ∇g if $g(x, y) = xy$. What can we say about g by looking at this plot?
3. If $\mathbf{F} = \langle x, yz, xz \rangle$, find $\operatorname{div} \mathbf{F}$ and $\operatorname{curl} \mathbf{F}$. What (very vaguely) do these tell us about \mathbf{F} ?
4. Find the potential function for the vector field $\mathbf{F} = \langle 2xyz, x^2y, x^2z \rangle$. For $\mathbf{G} = \langle 2xy + yz, xy^2, yz \rangle$?
5. Problems 11-14 on p.1021 of your textbook: match the gradient vector field with the function.
6. Suppose that F is a differentiable vector field and that f is a differentiable scalar function. Show that

$$\nabla \cdot (f\mathbf{G}) = f\nabla \cdot \mathbf{G} + (\nabla f) \cdot \mathbf{G}$$

7. Look again at your answers to Q.1: do these have nonzero divergence? nonzero curl? Now check by calculating.

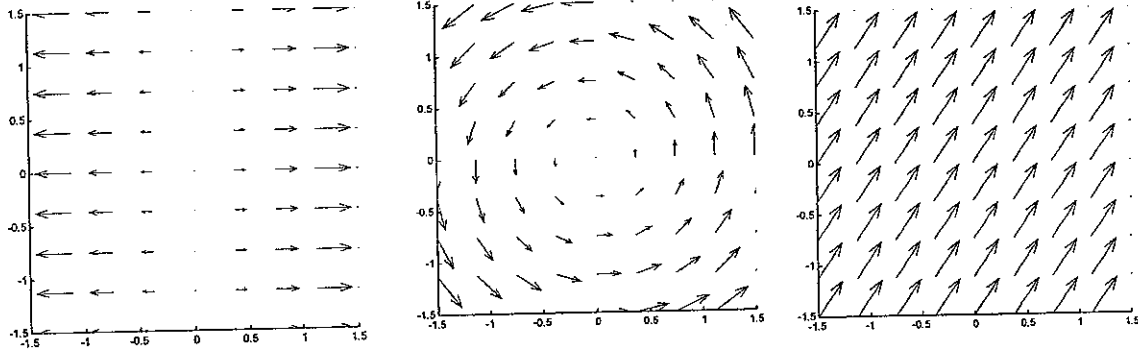
Warm-up for next time

Evaluate the line integral $\int_C f(x, y) ds$ where $f(x, y) = x^2 + y^2$ and C is the parametric curve $x = 4t - 1$, $y = 3t + 1$, $-1 \leq t \leq 1$.

Topic: Vector Calculus

Question: Which of the following vector fields cannot be a gradient vector field?

Image:



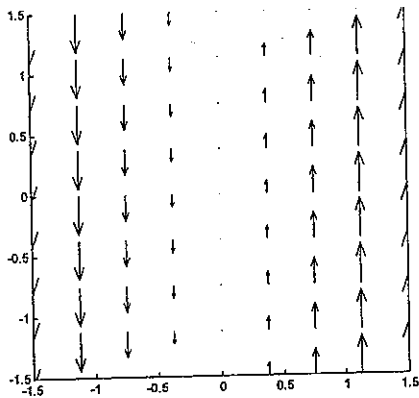
Choices:

- 1) the one on the left
- 2) the one in the middle
- 3) the the one on the right

Topic: Vector Calculus

Question: Which of the following could be a formula for the vector field pictured?

Image:



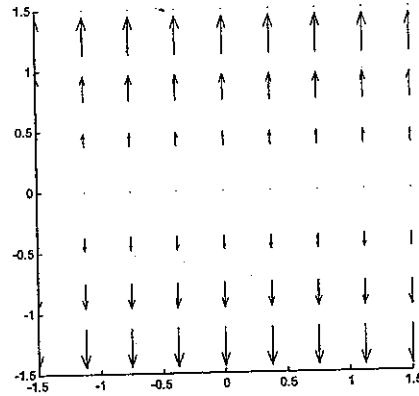
Choices:

- 1) $F(x, y) = x\mathbf{i}$
- 2) $F(x, y) = y\mathbf{j}$
- 3) $F(x, y) = x\mathbf{j}$
- 4) $F(x, y) = y\mathbf{i}$

Topic: Vector Calculus

Question: The function that describes the distance of a particle from the x-axis as it follows a flow line is:

Image:



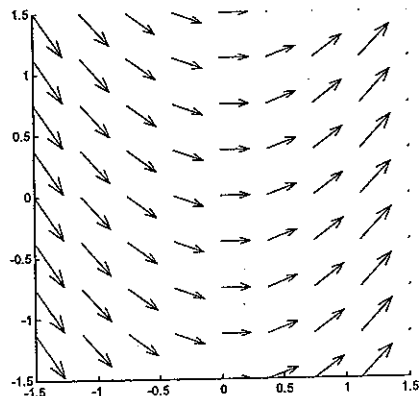
Choices:

- 1) linear
- 2) exponential
- 3) sinusoidal
- 4) logarithmic

Question: The flow lines for the vector field pictured will

be:

Image:



Choices:

- 1) straight lines
- 2) circles
- 3) ellipses
- 4) parabolas