Math 241, Spring 2007, Merit Worksheet 26

1. Evaluate the surface integral

$$\int\int_S y + z + 3 \, dS,$$

where S is the portion of the plane $z - 2x + 3y = 0$ that lies inside the cylinder $x^2 + y^2 = 4$.

2. Find the polar moment of inertia of the part of the surface $z = xy$ that lies inside the cylinder $x^2 + y^2 = 25$.

3. Evaluate the surface integral

$$\int\int_S 2x \, dy \, dz + 2y \, dz \, dx + 3 \, dx \, dy,$$

where S is the portion of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane with orientation given by the upward-pointing unit normal vector. Did you see two methods for approaching a problem like this in class? Can you check that both methods leave you with the same (ordinary) double integral to evaluate?

4. Evaluate the surface integral

$$\int\int_S x \, dy \, dz + y \, dz \, dx,$$

where S is the portion of the sphere $x^2 + y^2 + z^2 = 9$ that lies above the xy-plane with orientation given by the upward-pointing unit normal vector.

5. Evaluate the surface integral

$$\int\int_S \mathbf{F} \cdot \mathbf{n} \, dS,$$

where $\mathbf{n}$ is the upward-pointing unit normal vector to the surface S, the first octant part of the plane $2x + 2y + 2z = 3$ and $\mathbf{F} = x\vec{i} + y\vec{j} + z\vec{k}$.

6. Evaluate the surface integral

$$\int\int_S \mathbf{F} \cdot \mathbf{n}\, dS,$$

where $\mathbf{n}$ is the upward-pointing unit normal vector to the surface S, the part of the cone $z = r$ that lies within the cylinder $r = 3$ and $\mathbf{F} = y\vec{i} - x\vec{j}$.

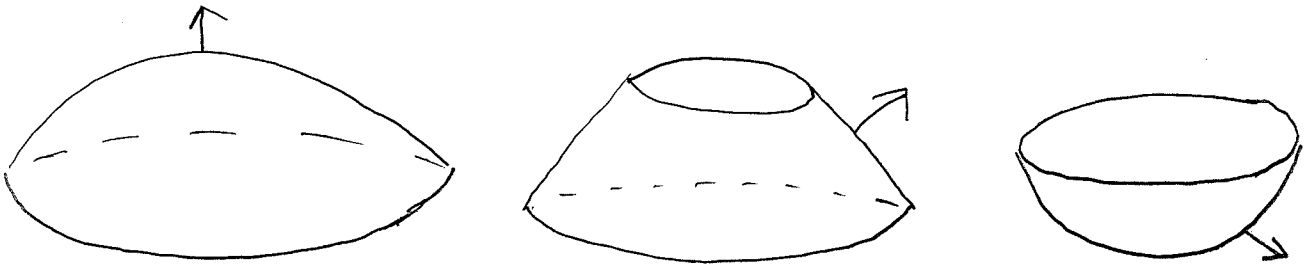
7. Suppose that $\mathbf{F} = (x^2+y^2+z^2)(x\vec{i}+y\vec{j}+z\vec{k})$ and that S is the spherical surface $x^2 + y^2 + z^2 = a^2$. Evaluate

$$\int\int_S \mathbf{F} \cdot \mathbf{n}\, dS,$$

without performing an antidifferentiation.

8. Calculate the outward flux of the vector field $\mathbf{F}(x, y, z) = 2x\vec{i} + 2y\vec{j} + 3\vec{k}$ across the closed surface S, the boundary of the solid paraboloid bounded by the xy-plane and $z = 4 - x^2 - y^2$.

9. Mark the induced orientations of the boundary curves:



Warm-up for next time

What does the Divergence Theorem say?

Review Thursday, May 3rd? Your opinions, please.