Math 241, Spring 2007, Merit Worksheet 27

1. Evaluate the surface integral
$$\int\int_S \mathbf{F}\cdot\mathbf{n}\,dS,$$
where $\mathbf{n}$ is the upward-pointing unit normal vector to the surface S, the first octant part of the plane $2x+2y+2z=3$ and $\mathbf{F}=x\vec{i}+y\vec{j}+z\vec{k}$.

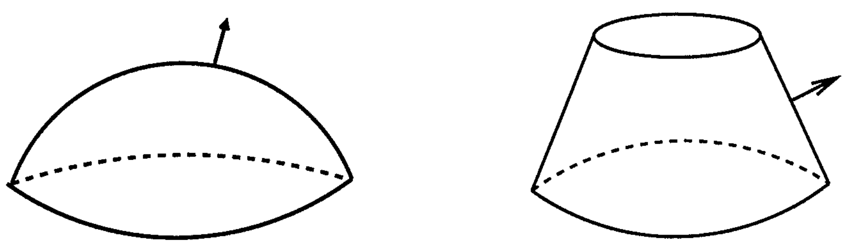
2. Evaluate the surface integral
$$\int\int_S \mathbf{F}\cdot\mathbf{n}\,dS,$$
where $\mathbf{n}$ is the upward-pointing unit normal vector to the surface S, the part of the cone $z=r$ that lies within the cylinder $r=3$ and $\mathbf{F}=y\vec{i}-x\vec{j}$.

3. Suppose that $\mathbf{F}=(x^2+y^2+z^2)(x\vec{i}+y\vec{j}+z\vec{k})$ and that S is the spherical surface $x^2+y^2+z^2=a^2$. Evaluate
$$\int\int_S \mathbf{F}\cdot\mathbf{n}\,dS,$$
without performing an antidifferentiation.

4. Calculate the outward flux of the vector field $\mathbf{F}(x,y,z)=2x\vec{i}+2y\vec{j}+3\vec{k}$ across the closed surface S, the boundary of the solid paraboloid bounded by the xy-plane and $z=4-x^2-y^2$.

5. Evaluate $\int\int_S \mathbf{F}\cdot\mathbf{n}\,dS$, where $\mathbf{F}=(x^3+e^z)\vec{i}+x^2y\vec{j}+(\sin xy)\vec{k}$ and S is the boundary of the region bounded by the parabolic cylinder $z=4-x^2$ and the planes $y=0$, $z=0$ and $y+z=5$.

6. Mark the induced orientations of the boundary curves:



7. Evaluate $\int\int_S (curl\ \mathbf{F}) \cdot \mathbf{n}\ dS$, where $\mathbf{F} =< yz, -xz, z^3 >$ and S is the part of the cone that lies between the two planes $z = 1$ and $z = 3$ and with upper unit normal vector n.

8. Evaluate $\int_C \mathbf{F} \cdot \mathbf{T}\ ds$, where $\mathbf{F} = y\vec{i} + z\vec{j} + x\vec{k}$ and C is the boundary of the triangle with vertices $(0, 0, 0)$, $(2, 0, 0)$ and $(0, 2, 2)$, oriented clockwise as viewed from above.

9. Problem 17, p.1073

10. Suppose that $f(x, y, z)$ has continuous second partial derivatives. What can you say about $curl\ \nabla f$? (Using Stokes' theorem, for example.)

    (a) $curl = 0$

    (b) $curl > 0$

    (c) $curl < 0$

    (d) Depends on f.

Warm-up for next time

Review Wednesday, May 2nd. I'll be on campus Thursday and Friday if you have any questions.

It has been a real pleasure working with you. Best of luck with all your exams.

Have a great Summer!