Math 241, Fall 2006, Merit Practice Exam 2

1. Let $f(x,y,z) = \sqrt{xy^2z^3}$ and let P be the point $(2,2,2)$.
   (a) Find the maximum directional derivative of f at P and the direction in which it occurs.
   (b) Find the directional derivative of f at P in the direction of $\vec{v} = 3\vec{i} + 12\vec{j} + 4\vec{k}$.

2. Find and classify the critical points of the function $f(x,y) = 4xy - 2x^4 - y^2$.

3. Find the first octant point on the surface $xyz = 8$ that is closest to $(0,0,0)$.

4. Find the equation of the tangent plane to the surface $xy^2 + 2xyz - e^{xz} = 8$ at the point $(1,3,0)$.

5. Show that the limit does not exist:
$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+y^3}$$

6. Show that the function
$$f(x,y) = \begin{cases} \frac{2x^6+3y^5}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$
is differentiable everywhere.

7. Use linear approximation to estimate
$$\sqrt{(3.1)^2 + (4.2)^2 + (11.7)^2}$$

8. Suppose that $w = f(u,v)$ and $u(x,y) = 2x + 3y$ and $v(x,y) = xy$. Find
$$\frac{\partial^2 w}{\partial x \partial y}$$
in terms of x, y and derivatives with respect to x and y.

9. Notice that this is (far) too long to be the real exam