Math 242, Merit Review Questions for Hour Exam 3, Fall 2006

Usual warnings apply: I am not writing the exam and I have not seen the exam. I would also suggest looking at the homework problems, *especially the extra problems posted online*. You should also look at the list of topics for exam 3 that Professor Nikolaev has posted online. Tuesday morning will be a review session for the exam.

1. Find the absolute maximum and minimum values that the function $f(x, y, z) = x^2 - yz$ takes on the region $x^2 + y^2 + z^2 \leq 1$.

2. Evaluate the integral

$$\int_0^4 \int_{\sqrt{y}}^2 \frac{ye^{x^2}}{x^3}\, dx\, dy.$$

3. Find the area of that section of the saddle-shaped surface $z = xy$ inside the cylinder $x^2 + y^2 = 1$.

4. Substitute $u = xy$ and $v = xy^3$ to find the area of the first quadrant region bounded by the curves $xy = 2$, $xy = 4$, $xy^3 = 3$, $xy^3 = 6$. (Fig. 13.9.8, p.1010)

5. Convert to rectangular coordinates: $\int_{\pi/2}^{3\pi/2} \int_{1/2}^{2} \frac{1}{r^3}\, dr\, d\theta$.

6. Convert to cylindrical coordinates: $\int_1^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} xy^2\, dy\, dx$.

7. Set up the integrals but do not evaluate:

   Consider a lamina that occupies the region D bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant with density function $\delta(x, y) = y$.

   (a) Find the mass of the lamina.

   (b) Find the centroid.

   (c) Find the moments of inertia around the x- and y-axes.

8. Set up but do not evaluate the triple integrals:

(a) the triple integral for the volume of the solid bounded by the planes $z = 0$, $z = 20 - x$ and the cylinder $x^2 + y^2 = 25$.

(b) the mass and centroid of a tetrahedron with density $\delta(x, y, z) = xy + z^2$, where the tetrahedron lies in the first octant, bounded by the coordinate axes and the plane $x + y + z = 1$.

(c) the triple integral for the volume of the solid bounded by $z = x^2$, $y + z = 4$, $y = 0$, $z = 0$.

9. Calculate the divergence and curl of the vector field:

$$\mathbf{F}(x, y, z) = xy^2\vec{i} + yz^2\vec{j} + zx^2\vec{k}.$$

10. Evaluate the line integral along the curve $y = x^3$ as x goes from $x = 3$ to $x = 0$.

$$\int_C xy^2\ dx\ +\ xy\ dy.$$

11. Evaluate the arclength integral along the curve $y = x^3$ as x goes from $x = 3$ to $x = 0$.

$$\int_C y\ ds.$$

12. Evaluate

$$\int_C F_t\ ds$$

where $\mathbf{F}(x, y, z) = y\vec{i} - x\vec{j} + z\vec{k}$ and C is the curve parametrized by $x = \sin t, y = \cos t, z = 2t$, for $0 \leq t \leq \pi$.