

Math 241, Merit Review Questions, Fall 2006

The usual warnings apply: this review sheet has no “predictive” value whatsoever but hopefully it will help you ask yourself “Do I remember that concept?” Best of luck in the final.

1. Write both symmetric and parametric equations for the line that passes through $P_1(1, -1, 2)$ and $P_2(3, 2, -1)$.
2. Given the four points $A(2, 3, 2)$, $B(4, 1, 0)$, $C(-1, 2, 0)$, and $D(5, 4, -2)$, find an equation of the plane that passes through A and B and is parallel to the line through C and D .
3. Write an equation for the plane through the point $(1, 1, 1)$ that is normal to the twisted cubic $x = t$, $y = t^2$, $z = t^3$ at this point.
4. At time $t = 0$, a ground target is 160 ft from a gun and is moving directly away from it with a constant speed of 80 ft/s. If the muzzle velocity of the gun is 320 ft/s, at what angle of elevation should it be fired in order to strike the moving target?
5. A particle moves in space with parametric equations $x = t$, $y = t^2$, $z = \frac{4}{3}t^{\frac{3}{2}}$. Find the curvature of its trajectory and the tangential and normal components of its acceleration when $t = 1$.
6. Find the vectors \mathbf{N} and \mathbf{T} at the point of the curve $x(t) = t \cos t$, $y(t) = t \sin t$ that corresponds to $t = \pi/2$.
7. Use spherical coordinates to show that

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^3 + y^3 - z^3}{x^2 + y^2 + z^2} = 0$$

8. Write an equation of the plane tangent to the surface

$$\sin xy + \sin yz + \sin xz = 1$$

at the point $(1, \pi/2, 0)$.

9. Each of the semiaxes a, b, c of an ellipsoid with volume $V = \frac{4}{3}\pi abc$ is measured with a maximum percentage error of 1%. Estimate the maximum percentage error in the calculated value of V .

10. Find the point of the surface $z = xy + 1$ that is closest to the origin.
11. Locate and classify the critical points of the function $f(x, y) = x^3y^2(1 - x - y)$
12. Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.
13. Find the directional derivative of $f(x, y) = 2\sqrt{x} - y^2$ at the point $(1, 5)$ in the direction towards the point $(4, 1)$. In what direction is the maximum rate of change?
14. If $z = \cos xy + y \cos x$, where $x = u^2 + v$ and $y = u - v^2$, use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$. z_{uu} ?
15. Use the definition of the limit to find

$$\lim_{(x,y) \rightarrow (0,0)} 3xy + 2x^2 + 2y^2 + 7$$

16. Investigate the differentiability of the function

$$f(x, y) = \frac{x^3y^2}{x^4 + y^4}.$$

17. Switch the order of integration and sketch the region of integration

$$\int_{-1}^3 \int_{x^2-4}^{2x-1} f(x, y) dy dx.$$

18. Find the volume of the solid bounded by the paraboloid $y = x^2 + 3z^2$ and the parabolic cylinder $y = 4 - z^2$.
19. Find the mass and centroid of the region bounded by $y = x^2$ and $x = y^2$, with density $\rho(x, y) = x^2 + y^2$.
20. Set up the integrals in both spherical and cylindrical coordinates to find the volume of the region bounded by $x^2 + y^2 + z^2 = 9$, $z = 0$ and the half-cone $z = \sqrt{2}\sqrt{x^2 + y^2}$.

21. Let R be the first quadrant region bounded by the circles $x^2 + y^2 = 2x$, $x^2 + y^2 = 6x$ and the circles $x^2 + y^2 = 2y$, $x^2 + y^2 = 8y$. Use the transformations

$$u = \frac{2x}{x^2 + y^2}, \quad v = \frac{2y}{x^2 + y^2}$$

to evaluate the integral

$$\int \int_R \frac{1}{(x^2 + y^2)^2} dx dy.$$

22. Let C be the curve parametrised by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ from the point $(1, 1, 1)$ to $(0, 0, 0)$. Evaluate the line integral

$$\int_C xy dx + yz dy + z^2 dz$$

23. Let C be the circle $x^2 + y^2 = 9$ and $\mathbf{F} = \langle -xy + \log \sin x, e^y \cos y^3 \rangle$. Evaluate the integral

$$\int_C \mathbf{F}_t ds.$$

24. Let C be the circle $x^2 + y^2 = 9$ and $\mathbf{F} = \langle -xy + e^y \cos y^3, \log \sin x + 3y \rangle$. Evaluate the integral

$$\int_C \mathbf{F}_n ds.$$

25. Evaluate the surface integral

$$\int \int_S y dy dz - x dz dx,$$

where S is the part of the cone $z = r$ that lies within the cylinder $r = 3$, where S has positive orientation given by its upward unit normal vector.

26. Evaluate the surface integral

$$\int \int_S \mathbf{F} \cdot \mathbf{n} da(S),$$

where S is the surface of the tetrahedron bounded by the three coordinate planes and the plane $x + y + z = 1$, oriented by its outer unit normal vector and $\mathbf{F} = \langle x^2 - \sin y^2 z, \log x z^2 + 2y, e^{xy} + 11y \rangle$.

27. Evaluate the integral

$$\int_C \mathbf{F}_t ds,$$

where C is the boundary of the part of the plane $x + 2y + z = 2$ that lies in the first octant, oriented counterclockwise as viewed from above and $\mathbf{F} = \langle y - x, x - z, x - y \rangle$.