

Math 241, Fall 2006, Merit Worksheet 9

1. Find the distance between the two planes:

$$\mathbf{a.} \quad 3x - 2y - z + 3 = 0 \quad -6x + 4y + 2z + 11 = 0$$

and

$$\mathbf{b.} \quad 2x - 2y - z + 3 = 0 \quad 5x + y + z = 9 = 0$$

2. Given the points $(0, 1, 1)$ and $(1, 1, -6)$ that lie on line 1 and points $(-4, 2, 1)$ and $(-1, 6, 2)$ that lie on line 2,
- Find the equation of the two lines.
 - Show that the two lines are perpendicular.
3. Find the length of the curve $\vec{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t \rangle$ as t goes from 0 to 1.
4. A particle starts at the origin with initial velocity $\vec{i} + 2\vec{j} + \vec{k}$. Its acceleration is $\vec{a}(t) = \langle t, t^2, t^3 \rangle$. Find its position vector.
5. Find the line of intersection of the planes in Q.1.b.
6. Let $\mathbf{a} = \langle 2, -5, 1 \rangle$, $\mathbf{b} = \langle -1, 3, 5 \rangle$ and $\mathbf{c} = \langle 2, 2, 3 \rangle$. Find
- $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$.
 - the angle between \mathbf{a} and \mathbf{b} .
 - two vectors perpendicular to both \mathbf{a} and \mathbf{b} .
 - a unit vector in the same direction as \mathbf{c} .
 - the area of the triangle with vertices $P(0, 0, 0)$, $Q(2, -5, 1)$ and $R(-1, 3, 5)$
 - the volume of the parallelepiped determined by \mathbf{a} , \mathbf{b} , and \mathbf{c} .
7. Newton's Second Law states: If at time t a force $\mathbf{F}(t)$ acts on an object of mass m producing an acceleration of $\mathbf{a}(t)$, then $\mathbf{F}(t) = m\mathbf{a}(t)$. Suppose a force with magnitude 15 N acts directly upwards from the xy -plane on an object with mass 5 kg . The object has an initial velocity $\mathbf{v}(0) = 2\mathbf{i} - \mathbf{j}$. Find the speed of the object at time $t = 2$.

8. Consider a moving particle with position vector $\mathbf{r}(t) = \langle \cos t^2, \sin t^2, 1/2 t^2 \rangle$ at time t . When $t = \sqrt{\pi/4}$, find:
- (a) the unit vectors \mathbf{N} and $\mathbf{T} = \bar{\mathbf{v}}$.
 - (b) the tangential and normal components of the acceleration at time t .
 - (c) the curvature κ .
 - (d) the osculating plane.
9. Find the osculating circle to $y = 7 - x^4$ at the point $(1, 6)$.

Warm-Up Problems for Next Time

1. Take the exam. This is section AD6 so you are in 151 Everitt for the exam.
2. Merit workshop will start at **10 am** this Thursday - an hour shorter as you will have spent an hour at the exam tonight.
3. Good Luck!