

## Math 241, Fall 2006, Merit Worksheet 12

- 1. Polar Coordinates:** Which of the following regions resembles a quarter of a doughnut?
  - (a)  $0 \leq r \leq 5, 0 \leq \theta \leq \pi/2$
  - (b)  $3 \leq r \leq 5, 0 \leq \theta \leq 2\pi$
  - (c)  $3 \leq r \leq 5, \pi \leq \theta \leq 2\pi$
  - (d)  $3 \leq r \leq 5, \pi \leq \theta \leq 3\pi/2$
- 2. Cylindrical Coordinates:**
  - (a) Graph  $r = 5$ .
  - (b) Graph  $\theta = 3\pi/4$ .
  - (c) Graph  $z = 7\pi/4$ .
  - (d) Mark the point  $(5, 3\pi/4, 7\pi/4)$ . What are its cartesian coordinates?
- Which of the following regions represents the portion of a cylinder of height 4 and radius 3 above the 3rd quadrant of the  $xy$  plane?
  - (a)  $1 \leq r \leq 3, 0 \leq z \leq 4, 0 \leq \theta \leq \pi/2$
  - (b)  $0 \leq r \leq 4, 0 \leq z \leq 4, \pi \leq \theta \leq 3\pi/2$
  - (c)  $0 \leq r \leq 4, 0 \leq z \leq 3, \pi \leq \theta \leq 3\pi/2$
  - (d)  $0 \leq r \leq 3, 0 \leq z \leq 4, 0 \leq \theta \leq \pi/2$
- 4. Spherical Coordinates:**
  - (a) Graph  $\rho = 5$ .
  - (b) Graph  $\phi = 3\pi/4$ .
  - (c) Graph  $\theta = 7\pi/4$ .
  - (d) Mark the point  $(5, 3\pi/4, 7\pi/4)$ . What are its cartesian coordinates?
  - (e) Graph  $\phi = \pi/2$ .
- Which of the following describes the bottom half of a sphere of radius 4 centred on the origin?
  - (a)  $0 \leq \rho \leq 4, \pi/2 \leq \phi \leq \pi, 0 \leq \theta \leq 2\pi$

(b)  $0 \leq \rho \leq 4, 0 \leq \phi \leq \pi/2, 0 \leq \theta \leq 2\pi$

(c)  $0 \leq \rho \leq 4, 0 \leq \phi \leq \pi, 0 \leq \theta \leq \pi$

(d)  $0 \leq \rho \leq 4, 0 \leq \phi \leq \pi, \pi \leq \theta \leq 2\pi$

6. Find the domains of the following functions (on  $\mathbb{R}^3$ ):

(a)  $f(x, y, z) = \sqrt{x - y}$

(b)  $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$

(c)  $f(x, y, z) = \frac{\log xyz}{xy^2 - xy}$

(d)  $f(x, y, z) = 4x^2y^4z^8 + z^2 + \sqrt{1 + x^2}$

Find where the function in (a) has value 4. Find where the function in (b) has value 0. Find where the function in (d) has value  $-1$ .

7. A solid ball of radius 2 is centred at the origin. A hole of radius 1 is drilled through the sphere with the axis of the hole lying on the  $z$ -axis. Describe the solid region that remains in

(a) cylindrical coordinates;

(b) spherical coordinates.

8. What are the cylindrical coordinates of a sphere centred at  $(0, 0, 2)$  of radius 3?

9. Convert the equation into both cylindrical and spherical coordinates:

(a)  $x^2 + y^2 = 2x$

(b)  $z = x^2 - y^2$

10. Describe the graph of the equation:

(a)  $\rho = 4 \cos \phi$ .

(b)  $\rho^3 - 4\rho = 0$ .

### Warm-Up Problems for Next Time

1. Find the largest possible domain of definition for the function

$$f(x, y) = \frac{xy}{x^2 - y^2}$$

2. Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{\cos(x^2 + y^2)}{1 - x^2 - y^2}$ .