

Math 241, Fall 2006, Merit Worksheet 13

1. The diagram below shows the level curves of a function.

What path will result in the greatest change in altitude? Which path is the steepest?

- (a) A to B
 - (b) A to C
 - (c) A to D
 - (d) All the same.
2. Can you think of two or more surfaces which have the following as their level curves?
 3. What are the level curves of the function $z = e^{\sin(x^2+y^2)}$?
 4. Sketch some typical level curves of the function $f(x, y) = y - x^2$.
 5. Describe the level surfaces of the function $f(x, y, z) = x^2 + y^2 - z^2$
 6. Determine whether the limits below exist:

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz}{x^2 + y^2 + z^2}$$

$$\lim_{(x,y,z,w) \rightarrow (0,0,0,0)} \frac{x^2 + y^2 + z^2 - w^2}{x^2 + y^2 + z^2 + w^2}$$

7. Discuss the continuity of the function

$$h(x, y, z) = \begin{cases} \frac{\sin(x^2-y^2)}{x^2-y^2} & \text{unless } x^2 = y^2 \\ 1 & \text{if } x^2 = y^2 \end{cases}$$

8. What are the cylindrical coordinates of a sphere centred at $(0, 0, 2)$ of radius 3?
9. Convert the equation into both cylindrical and spherical coordinates:
 - (a) $x^2 + y^2 = 2x$
 - (b) $z = x^2 - y^2$

10. Describe the graph of the equation:

(a) $\rho = 4 \cos \phi$.

(b) $\rho^3 - 4\rho = 0$.

Warm-Up Problems for Next Time

1. If $f(x, y) = x^2y + x^3y^2 + 4e^{xy}$, find its partial derivatives with respect to x and y .