

Math 241 Fall 2006 Merit Worksheet 14

1. Find the one point at which the plane tangent to the surface

$$z = x^2 + 2xy + 2y^2 - 6x + 8y$$

is horizontal. (Why do we know that the tangent plane exists?) Find the tangent plane to this surface at the point $(1, 2, 23)$.

2. Let $f(x, y, z, w) = w^2 \sin(x^2 + y^2 + z^2)$. Find

$$f_{xyzw}(\sqrt{\pi/12}, \sqrt{\pi/12}, \sqrt{\pi/6}, 1) \quad \text{and} \quad \left. \frac{\partial^2 f}{\partial x \partial w} \right|_{\sqrt{\pi/4}, 0, 0, 2}.$$

3. Consider $f(x, y, z) = x^2 \cos(y^3 + z^2)$.

- (a) Why do we know that $f_{zyyxxx} = 0$ without doing any computations?
- (b) Do we also know, without any computations, that $f_{xyzzz} = 0$?

4. Suppose $f_x(3, 4) = 5$, $f_y(3, 4) = -2$, and $f(3, 4) = 6$. Assuming the function is differentiable, what is the best estimate for $f(3.1, 3.9)$ using this information?

5. Use differentials to approximate $\sqrt{26} \sqrt[3]{28} \sqrt[4]{17}$.

6. Use differentials to approximate $e^{0.4} = e^{(1.1)^2 - (0.9)^2}$.

Now do this problem by finding the tangent plane to a surface.

7. The base radius r and the height h of a right circular cylinder are measured as 3 cm and 9 cm, respectively. There is a possible error of 1 mm in each measurement. Use differentials to estimate the maximum possible error in computing:

- (a) the volume of the cylinder;
- (b) the total surface area of the cylinder.

8. Four positive numbers, each less than 50 are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from the rounding.

Warm-Up for Next Time

Make a list of ALL the steps involved in finding the maximum and minimum values of the function $f(x)$ on the interval $[a, b]$.