Math 241 Fall 2006, Merit Worksheet 15

1. Find the absolute maximum and minimum values attained by the function $f(x, y) = 3x^2y - 3xy - x + 2$ on the triangle with vertices $(0, 0)$, $(0, 2)$ and $(4, 0)$.

2. Find the maximum and minimum values attained by the function $f(x, y) = xy^2$ on the circular disk $x^2 + y^2 \leq 3$.

3. The function $f(x, y) = x^3 + 12xy + y^4$ has

   (a) No global maximum or minimum

   (b) A global max but no global min

   (c) A global min but no global max

   (d) Both a global max and a global min

4. Suppose that the function f is continuous on the disk D bounded by the unit circle $x^2 + y^2 = 1$. Is it possible that $f(x, y)$ attains both its maximum and its minimum values on D at points of the boundary circle? Illustrate your answer with an example.

5. Consider $z = 2x^2 + 8xy + y^4$. Does this surface open upwards or downwards? What is its highest/lowest point?

6. Explain your answer using the multivariable chain-rule: A company sells regular widgets for \$4 apiece and premium widgets for \$6 apiece. If the demand for regular widgets is growing at a rate of 200 widgets per year and the demand for premium widgets is dropping at the rate of 80 per year, the company's revenue is

   (a) staying constant;

   (b) increasing;

   (c) decreasing.

7. Find $\frac{\mathrm{d}w}{\mathrm{d}t}$ both by using the chain rule and by expressing w as a function of t before differentiating, where

$$w = e^{-x^2-y^2}, \quad x = t, \quad y = \sqrt{t}$$

8. Prof Nikolaev's cardboard box factory has an order for open-topped boxes with a volume of 600 in^3. The material for the bottom of the box costs $6¢/in^2$ and the material for its sides costs $5¢/in^2$. What are the dimensions of the box that is most economical to manufacture?

9. Find the first octant point $P(x, y, z)$ on the plane $2x + 3y + z = 49$ which is closest to the point $Q(7, -7, 0)$.

10. Find the maximum possible product of three positive numbers whose sum is 120.

11. A very long rectangle of sheet metal has width L and is to be folded to make a rain gutter. Maximize its volume by maximizing the cross-sectional area, as shown.

Warm-Up for Next Time

1. Read section 12.5. Please, please do. (Note example 8).

2. Find $\frac{\mathrm{d}w}{\mathrm{d}t}$ both by using the chain rule and by expressing w as a function of t before differentiating, where

$$w = \frac{1}{u^2 + v^2}; \quad u = \cos 2t; \quad v = \sin 2t.$$