Math 241 Fall 2006, Merit Worksheet 16

1. Find the first octant point $P(x, y, z)$ on the plane $2x + 3y + z = 49$ which is closest to the point $Q(7, -7, 0)$.

2. Find the maximum possible product of three positive numbers whose sum is 120.

3. Use the example $x^2 + y^2 = 1$ to explain (in detail) the implicit function theorem.

4. Use implicit differentiation to find z_x and z_y where z is the function implicitly defined by $x - yz + xy^2z^3 = 1$ at the point $(1, 1, 1)$.

5. A particle Q moving through space is being studied. Let s denote the distance that Q has travelled with respect to some starting point. (We can think of distance as arc length on the curve determined by Q.)

   (a) We know that S depends on two factors X and Y.

   (b) We know that X and Y vary over time t in years according to the formulae $X = t^2 - 1$ and $Y = \ln t$.

   (c) The changes in s with respect to X and Y are both constants, a and b respectively.

   What is the speed of the particle Q, in terms of a and b, after 20 years?

6. Suppose $R = f(u, v, w)$, $u = g(x, y, z)$, $v = h(x, y, z)$ and $w = j(x, y, z)$. In the chain rule, how many terms do you have to add up to find the partial derivative with respect to x?

7. Suppose $w = \ln(x^2 + y^2 + z^2)$, where $x = s - t$, $y = s + t$ and $z = 2\sqrt{st}$. Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$.

8. If $g(s, t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfies the partial differential equation

$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0$$

9. The radius of a right circular cylinder is decreasing at a rate of 1.5 cm/s while its height is increasing at a rate of 4 cm/s. At what rate is the volume of the cylinder changing when the radius is 50 cm and the height is 100cm. The surface area?

10. If $f(u,v) = 2u^2v$ and $u(x,y) = x + 2y$ and $v = x^2 - y$, calculate f_{xx}.

11. Change variables in the partial differential equation $z_{xx} - z_{yy} - z_{xy} = 0$ if $u = x^2 + y^2$, $v = 2xy$.

Warm-Up for Next Time

1. Find the gradient vector to the function $f(x,y,z) = y^2 - z^2$ at the point $P(17,3,2)$.