Math 241 Fall 2006, Merit Worksheet 17

1. If $f(u, v) = 2u^2v$ and $u(x, y) = x + 2y$ and $v(x, y) = x^2 - y$, calculate f_{xy}, using the symbolic chain rule.

2. Let $f(x, y, z) = x \cos y + z^2$. Let P be the point $(3, \pi/3, 5)$.

   (a) Find $\nabla f(P)$.

   (b) Find the directional derivative of f in the direction $\vec{j}$.

   (c) Find the directional derivative of f in the direction $\vec{i} + \vec{j} + \vec{k}$.

3. What is the maximal value that a directional derivative of $f(x, y) = \frac{2x}{x-y}$ can assume at the point $(3, 1)$?

4. The temperature at a point (x, y) on a metal plate is given by $T(x, y) = 100 - x^2 - 2y^2$. A heat-seeking particle starts at the point $(4, 2)$ and moves at each instant in the direction of maximum temperature increase.

   (a) In what direction does the particle move initially?

   (b) Draw some level curves of T in the xy-plane and sketch the path followed by the particle.

   (c) What is the relationship between the tangent vectors to the curve and the gradient vectors of T?

5. Let f be a function of two variables that has continuous partial derivatives and consider the points $A(1, 3), B(3, 3), C(1, 7)$ and $D(6, 15)$. The directional derivative of f at A in the direction towards the point B is 3 and the directional derivative of f at A in the direction towards the point C is 9. What is the directional derivative of f at A in the direction of D?

6. Suppose that you are standing at the point with coordinates $(-100, -100, 430)$ on a hill that has the shape of the graph of $z = 500 - (0.003)x^2 - (0.004)y^2$ (in units of metres). In what (horizontal) direction should you move in order to maintain a constant altitude?

7. Use the contour plot on the next page (it shows the level curves of a surface) to answer the following questions: At which point will the

gradient vector have the largest magnitude? At which of these points will the gradient vector be most parallel to $\vec{j}$?

(a) $(0,4)$

(b) $(-4,-4)$

(c) $(0,0)$

(d) $(6,-2)$

8. Let $f(x,y,z) = x^2+y^2+z^2$. Which statement best describes the vector $\nabla\ g(x,y,z)$ and **why**? It is always perpendicular to:

   (a) a vertical cylinder passing through (x,y,z).

   (b) a horizontal plane passing through (x,y,z).

   (c) a sphere passing through (x,y,z).

   (d) none of the above.

Warm-Up for Next Time

1. Use the gradient vector to find the tangent plane to the surface $3z^3 = -x^2+4y^2$ at the point $(1,1,-1)$.