

Math 241 Fall 2006, Merit Worksheet 18

- Suppose that you are standing at the point with coordinates $(-100, -100, 430)$ on a hill that has the shape of the graph of $z = 500 - (0.003)x^2 - (0.004)y^2$ (in units of metres). In what (horizontal) direction should you move in order to maintain a constant altitude?
- Use the contour plot on the next page (it shows the level curves of a surface) to answer the following questions: At which point will the gradient vector have the largest magnitude? At which of these points will the gradient vector be most parallel to \vec{j} ?
 - $(0, 4)$
 - $(-4, -4)$
 - $(0, 0)$
 - $(6, -2)$
- Use the gradient vector to write an equation for the line tangent to the curve $x^4 + xy + y^2 - 19 = 0$ at the point $(2, -3)$.
- The surfaces $x^2y^2 + ax + z^3 = 16$ and $3x^2 + y^2 - 2z = 9$ intersect in a curve that passes through the point $P(2, 1, 2)$. Find a tangent vector to the curve of intersection at P .
- Find an equation for the plane tangent to the paraboloid $z = 2x^2 + 3y^2$ and, simultaneously, parallel to the plane $4x - 3y - z = 10$.
- The curve $\mathbf{r}(t) = \langle t^2/2, 4/t, t/2 - t^2 \rangle$ intersects the surface $x^2 - 4y^2 - 4z = 0$ at the point $(2, -2, 3)$. What is the angle of intersection?
- Consider the function $f(x, y) = \frac{1}{2}(ax^2 + by^2)$.
 - Show that $(0, 0)$ is a critical point.
 - For what values of a and b does f have a maximum at $(0, 0)$?
What does the surface look like?
 - For what values of a and b does f have a minimum at $(0, 0)$?
What does the surface look like?
 - For what values of a and b does f have a saddle point at $(0, 0)$?
What does the surface look like?

8. Which of the following guarantees a saddle point of the function $f(x, y)$ at the point (a, b) ?
- (a) f_{xx} and f_{yy} have the same sign at (a, b)
 - (b) f_{xx} and f_{yy} have different signs at (a, b)
 - (c) f_{xy} is negative at (a, b)
 - (d) None of the above.
9. Which of the following would be enough evidence to conclude that $f(x, y)$ has a global minimum?
- (a) D is always positive
 - (b) $f_{xx} > 0$ and $f_{yy} > 0$
 - (c) $f(x, y)$ has no saddle point and no local maxima
 - (d) None of the above.

Warm-Up for Next Time

1. Find and classify the critical points of the function

$$f(x, y) = x^3 + y^3 + 3xy + 3$$