

Math 241, Fall 2006, Merit Worksheet 2

1. The dot product of two vectors $\underline{a} = a_1\mathbf{i} + a_2\mathbf{j}$ and $\underline{b} = b_1\mathbf{i} + b_2\mathbf{j}$ in \mathbb{R}^2 is $a_1b_1 + a_2b_2$. It is written $\underline{a} \cdot \underline{b}$.

Let $\underline{a} = -4\mathbf{i} + 6\mathbf{j}$, $\underline{b} = 1\mathbf{i} + 3\mathbf{j}$, $\underline{c} = 3\mathbf{i} + 2\mathbf{j}$. Sketch these vectors on a single graph.

- (a) Calculate $\underline{a} \cdot \underline{b}$, $\underline{a} \cdot \underline{c}$.
- (b) Do you notice anything?
- (c) Find $\underline{b} \cdot \underline{b}$, $\underline{a} \cdot \underline{a}$.
- (d) Find $\|\underline{a}\|$, $\|\underline{b}\|$.

2. Let $\underline{a} = 3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $\underline{b} = \mathbf{i} + 6\mathbf{j} + \mathbf{k}$ and $\underline{c} = 4\mathbf{i} - 7\mathbf{j} + 2\mathbf{k}$. (We could also write $\underline{a} = \langle 3, 7, 2 \rangle$.)

- (a) Calculate $\underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c}$.
- (b) Calculate $(\underline{a} + \underline{b}) \cdot \underline{c}$.
- (c) Calculate $(2\underline{a}) \cdot \underline{b}$, $2(\underline{a} \cdot \underline{b})$.
- (d) Prove these patterns always hold.

3. For what values of t are $\langle t, 1, t-1 \rangle$ and $\langle 2, -4, 1 \rangle$ perpendicular?

4. Find the angle θ between the following pairs of vectors:

$12\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $8\mathbf{i} - 15\mathbf{j}$.

Find the angle between $\vec{x} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and

- (a) the x -axis.
- (b) the y -axis.
- (c) Find the unit vector $\frac{\vec{x}}{\|\vec{x}\|}$.
- (d) Quickly say what the angle between \vec{x} and the z -axis is. Explain.
- (e) What are the direction cosines of \vec{x} ?

5. A mass of 125 kilograms is suspended by two cables, as shown in the following diagram.

Find the tension in each cable.

6. Suppose that the horizontal and vertical components of the three vectors shown balance. How much work is done by the constant force \mathbf{F} (parallel to the inclined plane) in pulling the weight mg up the inclined plane a vertical height h ?
7. Suppose that $\vec{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\vec{b} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$.
- (a) Find $\text{comp}_{\vec{a}} \vec{b}$, the component of \vec{a} along \vec{b} . (Hint: What are ways of finding $\text{comp}_{\vec{a}} \mathbf{i}$?)
 - (b) Find $\text{proj}_{\vec{a}} \vec{b}$, the projection of \vec{a} onto \vec{b} .
 - (c) Draw a sketch showing all of these.

Warm-Up Problems for Next Time

- (a) If $\vec{a} = \langle 5, 1, 4 \rangle$ and $\vec{b} = \langle -1, 0, 2 \rangle$, calculate $\vec{a} \times \vec{b}$.
- (b) Calculate $(\vec{a} \times \vec{b}) \cdot \vec{a}$.
- (c) Calculate $\text{comp}_{\vec{a}} \vec{b}$.