Math 241, Fall 2006, Merit Worksheet 3

1. You are given a line $ax + by + c = 0$. Find a normal vector $\vec{n}$ to this line.

   Hint: If $P_0 = (x_0, y_0)$ and $P_1 = (x_1, y_1)$ are points on the line, then $\vec{P_0P_1}$ is . . .

   This will be useful in one of your homework problems.

2. Calculate $(-\mathbf{i} + 4\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$.

3. Find the area of the triangle in $\mathbb{R}^3$ between the points $A = (1, 1, 1)$, $B = (4, 5, 2)$ and $C = (6, 3, 8)$.

4. The cross product:

   (a) Consider the vector $\vec{c} = \vec{a} \times (\vec{a} \times \vec{b})$. Is $\vec{c} \perp \vec{a}$? Is $\vec{c} \perp \vec{b}$? Prove or give a counterexample.

   (b) Suppose that $\mathbf{j} \times \vec{a} = \mathbf{i}$. Give two possible solutions for $\vec{a}$ and discuss the other possible solutions.

5. When are two vectors parallel? Draw pictures that explain and also make up a (nontrivial) example of two vectors that are parallel and two vectors that are not parallel.

6. When are two vectors perpendicular? Draw pictures and work out examples.

7. Given the points $A = (1, -2, 7)$ and $P = (x, y, z)$ and the vector $\vec{n} =< 1, -5, 3 >$:

   (a) Write an equation that says the following: $\vec{n}$ and $\vec{AP}$ are perpendicular.

   (b) Describe the set of points $P(x, y, z)$ that satisfy the equation from (a).

8. A parallelepiped is a 3-d object with parallelograms for side, like a tilted box. Its volume is the area of its base times its height (in the direction perpendicular to that base). The three vectors $\vec{a} = 6\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\vec{b} = \mathbf{i} + 5\mathbf{j}$, $\vec{c} = 2\mathbf{i} + 2\mathbf{j} + 7\mathbf{i}$ determine a parallelepiped. Find its volume. Find a formula for the volume of a parallelepiped with sides $\vec{x}, \vec{y}, \vec{z}$.

9. (a) Find the parametric and symmetric equations of the line through the points $P(-1, 0, 2)$ and $Q(2, 1, 1)$.

   (b) Find the midpoint of PQ and check that it satisfies the equation of the line.

10. Find the equation of the plane through the points $P(7, 2, 1)$, $Q(6, -1, 3)$ and $R(9, 3, 2)$.

Warm-Up Problems for Next Time

1. Find the equation of the plane through $(1, -3, 2)$ with normal vector $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$.

2. Find $\mathbf{r}'(0)$ for $\mathbf{r}(t) = e^{2t}\mathbf{i} + e^{-t}\mathbf{j}$.