

## Math 241, Fall 2006, Merit Worksheet 4

1. Let  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$  be vectors. Then

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Can you give a reason why  $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| = |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|$  holds (without doing any calculations)?

2. (a) Find the parametric and symmetric equations of the line through the points  $P(-1, 0, 2)$  and  $Q(2, 1, 1)$ .  
(b) Find the midpoint of  $PQ$  and check that it satisfies the equation of the line.
3. Find the equation of the plane through the points  $P(7, 2, 1)$ ,  $Q(6, -1, 3)$  and  $R(9, 3, 2)$ .
4. Find the angle  $\theta$  between the planes with equations

$$3x - 12y + 4z = 12 \quad \text{and} \quad 7x - 4y - z = 11$$

Write symmetric equations for their line of intersection  $L$ .

5. Let the plane  $H$  be given by the equation  $x + y + z = 10$ . Write down the equation of a sphere which is tangent to the plane at some point.
6. Show that the points  $A(1, 0, 3)$ ,  $B(3, -2, 9)$ ,  $C(-2, -3, 0)$ ,  $D(4, 1, -1)$  are coplanar.
7. Find the perpendicular distance between the point  $(-1, 2, 3)$  and the plane  $3x - 4y + z = 2$ .
8. A line  $L$  has symmetric equations

$$\frac{2x - 5}{1} = \frac{y - 3}{-2} = \frac{z + 4}{3}$$

- (a) By considering just the first inequality what do we obtain?  
(b) Similarly find two more. Is there anything special about them?

- (c) What is the connection between these and line  $L$ ?
9. What cases can arise when you have two lines  $L_1$  and  $L_2$  in the  $xy$ -plane? What does this tell us about two linear equations  $a_1x + b_1y = c_1$  and  $a_2x + b_2y = c_2$  in two unknowns  $x$  and  $y$ ?
10. What are the possible configurations for
- (a) Three lines in  $\mathbb{R}^3$ ?
  - (b) Three planes in  $\mathbb{R}^3$ ?

What does this tell you about solutions of suitable systems of linear equations?

### Warm-Up Problems for Next Time

1. Show that the graph of the curve with parametric equations  $x = t \sin 6t$ ,  $y = t \cos 6t$ ,  $z = t$  lies on the cone  $z = \sqrt{x^2 + y^2}$  with its vertex at the origin and opening upwards.
2. Suppose  $\mathbf{r}(t) = 3\mathbf{i} \cos 2\pi t + 3\mathbf{j} \sin 2\pi t$ . Find  $\mathbf{r}'(7/4)$ ,  $\mathbf{r}''(7/4)$ .