Math 241, Fall 2006, Merit Worksheet 5

1. Find the equation of the plane through $P(1,-1,1)$ that intersects the xy-plane in the same line as does the plane $3x+2y-z=6$.

2. Two nonparallel non-intersecting lines are said to be **skew lines**. Show that the lines L_1 and L_2 given below are skew lines:

$$L_1: \quad x=4+2t, \quad y=-5+4t, \quad z=1-3t$$
$$L_2: \quad x=2+t, \quad y=-1+3t, \quad z=2t$$

3. Find equations of the planes that are parallel to the plane $x+2y-2z=1$ and two units away from it.

4. Let $\vec{a},\vec{b},\vec{c}$ be three vectors in the plane $3x-5y+6z=7$. Compute $(-\vec{a}+4\vec{b}-7\vec{c})\cdot(-3\mathbf{i}+5\mathbf{j}-6\mathbf{k})$.

5. A particle has position vector $\mathbf{r}(t) =< 2-t, 1+t^2 >$. Sketch the path traced by this particle as t increases from -2 to 2. Label the points $\mathbf{r}(-2), \mathbf{r}(-1), \mathbf{r}(0), \mathbf{r}(1), \mathbf{r}(2)$. Find an equation for this curve using cartesian coordinates.

6. Show that the graph of the curve with parametric equations $x=t, y=\sin 5t, z=\cos 5t$ lies on the circular cylinder $y^2+z^2=1$ centered along the x-axis. Can you (roughly) graph or describe this curve?

7. Which of the following is not a parametrization of the entire curve $y=x^3$?

   (a) $x(t)=t; \quad y(t)=t^3,$
   (b) $x(t)=t^2; \quad y(t)=t^6,$
   (c) $x(t)=t^3; \quad y(t)=t^9,$
   (d) $x(t)=2t; \quad y(t)=8t^3.$

   What are the differences between those that do parameterize the curve $y=x^3$?

8. Which of the following best describes the path of a particle defined by the parametric equations

$$x(t)=\cos t^2, \quad y(t)=\sin t^2?$$

(a) a circle on which the particle moves faster and faster,

(b) a parabola on which the particle travels at constant speed,

(c) a parabola on which the particle travels faster and faster,

(d) a circle on which the particle moves slower and slower.

Come up with parameterizations of the other options.

9. Find the values of $\mathbf{r}'(1), \mathbf{r}''(1)$ and $v(1)$ for

$$\mathbf{r}(t) =< t^3, 2t^2 + 1 >$$

10. A particle has position vector $\mathbf{r}(t)$ and speed $v(t)$. Prove or disprove: Its scalar acceleration is $\frac{\mathrm{d}v}{\mathrm{d}t}$.

Warm-Up Problems for Next Time

1. Calculate $\int_0^{\pi/4} (\mathbf{i} \sin t + 2\mathbf{j} \cos t)\ \mathrm{d}t$.
2. List the connections between $\mathbf{r}(t), \mathbf{v}(t), \mathbf{a}(t), v(t), a(t)$.
3. Write out the product rules for differentiation.