Math 241, Fall 2006, Merit Worksheet 6

1. A lighthouse at position L is in the middle of a lake. Its beam is turning counterclockwise with constant angular velocity. At which point

   (a) is the velocity vector of the beam largest?

   (b) is the velocity vector of the beam most parallel to $\mathbf{j}$?

   (c) is the acceleration vector of the beam most parallel to $\mathbf{j}$?

2. Suppose that a point moves along the surface of a sphere. Show that its position vector is always perpendicular to its velocity vector.

3. Give an example of a curve in $\mathbb{R}^2$ or $\mathbb{R}^3$ for which the position at time $t = 10$ is the same as the position at time $t = 0$ but for which the speed is never 0.

   Can you find a differentiable function of a single variable with this property, i.e., a function $y = f(x)$ for which $f(10) = f(0)$ and $f'(x) \neq 0$ for $x \in [0, 10]$? Why or why not?

4. Find a parameterization of the curve formed by the intersection of the two surfaces:

$$x^2 + y^2 = 4 \qquad \text{and} \qquad x + y + z = 1$$

5. Write the curve $\mathbf{r}(t) =< 5t, 1 + t^3, 1 - 2t^2 >$ as the intersection of two surfaces.

6. Find the vector, parametric and symmetric equations for the tangent line to $\mathbf{r}(t) =< t^2, -2t, 1 + t^3 >$ at the point $(1, 2, 0)$.

   Find a tangent vector of length 2 at $(4, -4, 9)$.

7. Jethro throws a mango off a cliff and into the sea below. The cliff is 200m high and the mango is thrown with initial velocity $< 20, 30 >$ (measured in m/s). What is the greatest height the mango reaches? How much time passes before the fruit hits the waves? If Jethro changes his mind and decides to eat the mango, how far out will he have to swim to retrieve the floating mango? (Ignore air resistance but don't ignore gravity.)

8. A particle has position vector $\mathbf{r}(t)$ and speed $v(t)$. Prove or disprove: Its scalar acceleration is $\frac{\mathrm{d}v}{\mathrm{d}t}$.

Warm-Up Problems for Next Time

1. Find the arc-length of

$$x = 3\sin 2t, \quad y = 3\cos 2t, \quad z = 8t$$

from $t = 0$ to $t = \pi$.

2. Read pages 832-834. Sketch a cylinder and show some of its traces. On a separate sketch show some of its rulings.