

## Math 241, Fall 2006, Merit Worksheet 7

1. Consider the curve  $\mathbf{r}(t) = \sin t\mathbf{i} + t\mathbf{j} + \cos t\mathbf{k}$ ,  $0 \leq t \leq 2\pi$ .
  - (a) Find the arc length of  $\mathbf{r}(t)$ .
  - (b) Find the arc length of  $\mathbf{s}(t) = \sin t^2\mathbf{i} + t^2\mathbf{j} + \cos t^2\mathbf{k}$ ,  $0 \leq t \leq \sqrt{2\pi}$ .
  - (c) Find the arc length of  $\mathbf{w}(t) = \cos t\mathbf{i} + (t + \frac{\pi}{2})\mathbf{j} - \sin t\mathbf{k}$ ,  $-\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$ .
  - (d) Is there a relationship between  $\mathbf{r}$ ,  $\mathbf{s}$  and  $\mathbf{w}$ ?
  - (e) What does this tell us about arc length?
2. Find the arc-length parametrization of the helix

$$x(t) = 3 \cos t, \quad y(t) = 3 \sin t, \quad z(t) = 4t$$

in terms of the arc length  $s$  measured from the initial point  $(3, 0, 0)$ .

3. Sketch or describe the traces of  $x^2 - y^2 + z^2 = 1$  in both horizontal and vertical planes parallel to the coordinate axes. This is a *hyperboloid of one sheet* opening along the  $y$ -axis.
4. **True/False:** The equation for the cylinder through the circle  $x^2 + y^2 = 4$  is  $z = x^2 + y^2 - 4$ .  
What are the horizontal traces and the rulings? What are their equations?
5. Show that the projection into the  $xy$ -plane of the intersection of the plane  $z = y$  and the paraboloid  $z = x^2 + y^2$  is a circle.
6. Write an equation for the surface generated by rotating the curve  $y^2 + z^2 = 1$  about the  $z$ -axis. What are its traces parallel to the coordinate planes?
7. Sketch/ describe the traces of  $x^2 - y^2 - 9z^2 = 9$ . This is a *hyperboloid of two sheets* opening along the  $x$ -axis.

## Warm-Up Problems for Next Time

1. Find the curvature of

$$x = 3 \cos t, \quad y = 4 \sin t$$

where  $t = \pi/4$ .

2. Read Section 11.7. Make a list of ALL quadric surfaces. Your list should include the equation, name and a basic sketch of each surface. (This is something you are eventually going to have to memorise).