Math 241, Fall 2006, Merit Worksheet 8

1. What is the curvature at the points marked in the diagram?

2. Using the example of Problem 1, discuss the following terms: radius of curvature of a function, center of curvature, osculating circle (or circle of curvature). Find these for the point $(3/2, 3/2)$ on the folium of Descartes, the curve defined by $x^3 + y^3 = 3xy$. Sketch these on the graph

3. Ask me for a piece of aluminum foil. Yes, that's right - aluminum foil.

   Figure out what directions the unit tangent, normal and binormal vectors point in. What is the osculating plane? Find the osculating circle. Where is the curvature least? Greatest?

4. Consider the curve, $\mathbf{r}(t) =< e^{-t}\cos t, e^{-t}\sin t, e^{-t} >$, $t \geq 0$.

   (a) Graph the curve and prove that the curve exists on the surface $z^2 = x^2 + y^2$.

   (b) Find the curvature.

   (c) What happens to the curvature as $t \to \infty$?

   (d) Prove that as $t \to \infty$, the total distance travelled is $\sqrt{3}$.

5. Find the tangential and normal components of the acceleration for the curve $\mathbf{r}(t) =< t, t^2, t^3 >$. What is the curvature at the point $(2, 4, 8)$? What are the unit tangent, normal and binormal vectors at $(2, 4, 8)$? The plane of osculation?

6. Using tangential and normal components of acceleration, describe in words those curves $\mathbf{r}(t)$ for which

   (a) $\mathbf{a} \perp \mathbf{v}$

   (b) $\mathbf{a} \,||\mathbf{v}$

      i. Assuming initial position r_0 and initial velocity v_0, find an equation for the velocity of $\mathbf{v}$.

      ii. Find an equation for the position $\mathbf{r}(t)$.

iii. Describe the line which contains the range of the position vector $\mathbf{r}(t)$.

Warm-Up Problems for Next Time

1. Before leaving, let me know your preference for when to hold a practice exam.

2. Write out formulae for:

   curvature, unit tangent, normal and binormal vectors, tangential and normal components of acceleration

3. Note that my office hours are now:

   Monday 4.00-4.50 in 173 Altgeld

   and

   Thursday 11.00-11.50 in 173 Altgeld

   (or make an appointment by email).