Math 241, Fall 2006, Merit Worksheet 9

1. Find the curvature function $\kappa(x)$ of the function $y = x^4$. Draw rough sketches of both the curvature function and the curve itself.

2. Find the curvature of $x = t, y = 4t^{3/2}, z = -t^2$ at the point $(1, 4, -1)$.

3. Where does $y = e^x$ have maximum curvature?

4. Find the osculating circle to the curve $xy = 4$ at the point $(1, 4)$.

5. If $\mathbf{r}(t) =< t^2, 2/3t^3, t >$, find the vectors $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$ at $(1, 2/3, 1)$.

6. Find the tangential and normal components of the acceleration for the curve $\mathbf{r}(t) =< t, t^2, t^3 >$. What is the curvature at the point $(2, 4, 8)$? What are the unit tangent, normal and binormal vectors at $(2, 4, 8)$? The plane of osculation?

7. Using tangential and normal components of acceleration, describe in words those curves $\mathbf{r}(t)$ for which

   (a) $\mathbf{a} \perp \mathbf{v}$

   (b) $\mathbf{a} \,||\mathbf{v}$

      i. Assuming initial position r_0 and initial velocity v_0, find an equation for the velocity of $\mathbf{v}$.

      ii. Find an equation for the position $\mathbf{r}(t)$.

      iii. Describe the line which contains the range of the position vector $\mathbf{r}(t)$.

8. The angle between the vectors $-x\vec{i} - \vec{j} + \vec{k}$ and $x\vec{i} + 2\vec{j} - 3\vec{k}$:

   (a) is between 0 and 45 degrees

   (b) is between 45 and 90 degrees

   (c) is greater than 90 degrees

   (d) can be any of the above depending on the value of x.

9. Two vectors have a dot product of 14. To guarantee the dot product is equal to 28, you could:

   (a) double the angle between the vectors

(b) double the length of both vectors

(c) double the length of one vector

(d) none of the above

10. Which of the following is a point in the plane parallel to $3x+4y-2z=6$ containing the origin?

    (a) $(1,1,1)$

    (b) $(1,2,3)$

    (c) $(3,2,1)$

    (d) $(3,4,-2)$

    (e) None of the above.

11. (Do this without doing any calculations) In $\mathbb{R}^2$, consider the vector $\vec{v} = 5\vec{i} + 7\vec{j}$. For which unit vector below will the component of $\vec{v}$ perpendicular to that unit vector be largest?

    (a) $\vec{i}$

    (b) $(1/\sqrt{2})(\vec{i} - \vec{j})$

    (c) $\vec{j}$

    (d) $(1/\sqrt{2})(\vec{i} + \vec{j})$

12. The value of $(\vec{v} \times \vec{w}) \cdot \vec{w}$ is

    (a) $\vec{v} \cdot ||w||^2$

    (b) 0

    (c) $\vec{v} \times (\vec{w} \cdot \vec{w})$

    (d) $(\vec{v} \cdot \vec{w}) \times \vec{w}$

Warm-Up Problems for Next Time

1. Study for your exam!! (Tuesday at 7pm)

2. Practice Exam: Sunday at 5pm. (Probably in 173 Altgeld)

3. Office hours: Mon 4.00-4.50 in 173 Altgeld and Thurs 11.00-11.50 in 173 Altgeld (or make an appointment by email).