Math 242, Merit Practice Hour Exam 3, Fall 2005

1. Evaluate $\int_0^8 \int_{x^{2/3}}^4 x \cos y^4 \; dy \; dx$.

2. Find the volume of the solid bounded by the paraboloids $z = 2x^2 + 2y^2$ and $z = 48 - x^2 - y^2$.

3. Convert to rectangular coordinates: $\int_{\pi/2}^{3\pi/2} \int_{1/2}^{2} \frac{1}{r^3} \; dr \; d\theta$.

4. Convert to cylindrical coordinates: $\int_1^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} xy^2 \; dy \; dx$.

5. Apply the first Theorem of Pappus to find the volume of the solid obtained by rotating the region L around the x-axis, where L is the region in the first quadrant bounded by $x = 1$, $x = 3$ and $y = x^2$.

6. Apply the second Theorem of Pappus to find the surface area of the surface obtained by rotating the curve C around the y-axis, where C is the semi-circular arc drawn on the board. If C were more complicated, how would we find its arc-length?

   (C is the right half of the circle of radius 1, centred at $(3, 1)$.)

7. Set up the integrals but do not evaluate:

   Consider a lamina that occupies the region D bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant with density function $\delta(x, y) = y$.

   (a) Find the mass of the lamina.
   (b) Find the centroid.
   (c) Find the moments of inertia and radii of gyration around the x- and y-axes.

8. Set up but do not evaluate the triple integrals:

   (a) the triple integral for the volume of the solid bounded by the planes $z = 0$, $z = 20 - x$ and the cylinder $x^2 + y^2 = 25$.
   (b) the mass and centroid of a tetrahedron with density $\delta(x, y, z) = xy + z^2$, where the tetrahedron lies in the first octant, bounded by the coordinate axes and the plane $x + y + z = 1$.
   (c) the triple integral for the volume of the solid bounded by $z = x^2$, $y + z = 4$, $y = 0$, $z = 0$.