Math 242 Practice Exam 2

Usual caveats apply.

1. Let $f(x, y, z) = \sqrt{xy^2z^3}$ and let P be the point $(2, 2, 2)$.
   (a) Find the maximum directional derivative of f at P and the direction in which it occurs.
   (b) Find the directional derivative of f at P in the direction of $\mathbf{v} = 3\mathbf{i} + 12\mathbf{j} + 4\mathbf{k}$.
2. Find and classify the critical points of the function $f(x, y, z) = x^3 + 6xy + 3y^2$.
3. The plane $4x+9y+z = 0$ intersects the elliptic paraboloid $z = 2x^2+3y^2$ in an ellipse. Find the highest and lowest points on this ellipse.
4. Find the maximum and minimum values of the function $f(x, y) = x^2 + y^2 - 2x - y$ on the triangle with vertices $(0, 0), (0, 2)$, and $(2, 0)$.
5. Find the equation of the tangent plane to the surface $xy^2+2xyz-e^{xz} = 8$ at the point $(1, 3, 0)$.
6. Explain why the limit $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+y^3}$ does not exist.
7. Let $w = uv - xy$ and $u = \frac{x}{x^2+y^2}$ and $v = \frac{y}{x^2+y^2}$. Use the chain rule to find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ as functions of x and y.
8. The aggregate resistance R of three variable resistancs R_1, R_2, and R_3 connected in parallel satisfies the harmonic equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Suppose that R_1 and R_2 are 100Ω and are increasing at $1\Omega/s$ while R_3 is 200Ω and is decreasing at $2\Omega/s$. Is R increasing or decreasing at that instant? At what rate?
9. Use linear approximation to find $\sqrt{(3.1)^2 + (4.2)^2 + (11.7)^2}$.