Math 242, Merit Review Questions, Fall 2005

Here are some questions intended as a (partial, incomplete) review of the topics covered this semester. In particular, it only covers Chapters 11 and 12, and NOT chapter 13. Some of these questions are hard. I am also going to give you a photocopy of a list of review questions from a previous year.

1. Write both symmetric and parametric equations for the line that passes through $P_1(1, -1, 2)$ and $P_2(3, 2, -1)$.

2. Given the four points $A(2, 3, 2)$, $B(4, 1, 0)$, $C(-1, 2, 0)$, and $D(5, 4, -2)$, find an equation of the plane that passes through A and B and is parallel to the line through C and D.

3. Write an equation for the plane through the point $(1, 1, 1)$ that is normal to the twisted cubic $x = t$, $y = t^2$, $z = t^3$ at this point.

4. At time $t = 0$, a ground target is 160 ft from a gun and is moving directly away from it with a constant speed of 80 ft/s. If the muzzle velocity of the gun is 320 ft/s, at what angle of elevation should it be fired in order to strike the moving target?

5. A particle moves in space with parametric equations $x = t$, $y = t^2$, $z = \frac{4}{3}t^{\frac{3}{2}}$. Find the curvature of its trajectory and the tangential and normal components of its acceleration when $t = 1$.

6. The right branch of the hyperbola $x^2 - y^2 = 1$ may be parametrized by $x(t) = \cosh t$, $y(t) = \sinh t$. Find the point where its curvature is minimal.

7. Find the vectors (what did he use for notation here?) $\mathbf{N}$ and $\mathbf{T}$ at the point of the curve $x(t) = t\cos t$, $y(t) = t\sin t$ that corresponds to $t = \pi/2$.

8. Use spherical coordinates to show that

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x^3 + y^3 - z^3}{x^2 + y^2 + z^2} = 0$$

9. Prove that there is no function f with continuous second-order partial derivatives such that $f_x(x, y) = 6xy^2$ and $f_y(x, y) = 8x^2y$.

10. Write an equation of the plane tangent to the surface

$$\sin xy + \sin yz + \sin xz = 1$$

at the point $(1, \pi/2, 0)$.

11. You must build a rectangular shipping crate with volume $60ft^3$. Its sides cost $\$1/ft^2$, its top costs $\$2/ft^2$, and its bottom costs $\$3ft^2$. What dimensions would minimize the total cost of the box?

12. Each of the semiaxes a,b,c of an ellipsoid with volume $V = \frac{4}{3}\pi abc$ is measured with a maximum percentage error of 1%. Estimate the maximum percentage error in the calculated value of V.

13. Find the point of the surface $z = xy + 1$ that is closest to the origin.

14. Locate and classify the critical points of the function $f(x, y) = x^3y^2(1 - x - y)$

15. Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.

16. Find the directional derivative of $f(x, y) = 2\sqrt{x} - y^2$ at the point $(1, 5)$ in the direction of the point $(4, 1)$. In what direction is the maximum rate of change?

17. If $z = \cos xy + y \cos x$, where $x = u^2 + v$ and $y = u - v^2$, use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.