Merit Worksheet 12 - Math 242, Fall 2005

Review for Hour Exam 1

1. Let $\mathbf{a} =< 2, -5, 1 >$, $\mathbf{b} =< -1, 3, 5 >$ and $\mathbf{c} =< 2, 2, 3 >$. Find

   (a) $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$.

   (b) the angle between $\mathbf{a}$ and $\mathbf{b}$.

   (c) two vectors perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

   (d) a unit vector in the same direction as $\mathbf{c}$.

   (e) the area of the triangle with vertices $P(0, 0, 0)$, $Q(2, -5, 1)$ and $R(-1, 3, 5)$

   (f) the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

2. Find the work done by a force of 20 lbs acting in the direction of $\mathbf{v} =< 1, 1, 1 >$ in moving an object 4 feet along the positive y-axis.

3. Newton's Second Law states: If at time t a force $\mathbf{F}(t)$ acts on an object of mass m producing an acceleration of $\mathbf{a}(t)$, then $\mathbf{F}(t) = m\mathbf{a}(t)$. Suppose a force with magnitude 15 N acts directly upwards from the xy-plane on an object with mass 5 kg. The object has an initial velocity $\mathbf{v}(0) = 2\mathbf{i} - \mathbf{j}$. Find the speed of the object at time $t = 2$.

4. Sketch the graph of the equation $4x^2 - 3y^2 + 2z^2 = 1$. Label two points on your graph and verify that they lie on the surface. What are the traces parallel to the xy-,xz-,yz-planes?

5. Consider a moving particle with position vector $\mathbf{r}(t) =< e^t \cos t, e^t \sin t, e^t$ at time t.

   (a) Find the unit vectors $\mathbf{N}$ and $\mathbf{T} = \overline{\mathbf{v}}$.

   (b) Find the tangential and normal components of the acceleration at time t.

   (c) Find the curvature κ.

6. A torus (or doughnut shaped surface) is obtained by rotating a circle of radius b centred at $(a, 0)$ in the yz-plane around the z-axis. Write a radical-free equation describing this surface in

(a) cartesian coordinates.

(b) spherical coordinates.

(c) cylindrical coordinates.

Warm-Up for Thursday

Find the limit

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{6 - xy}{4 - \sqrt{xy + 10}}$$

where $f(x,y)$ is defined on $xy + 10 \geq 0$, $xy \neq 6$.