

Merit Worksheet 13 - Math 242, Fall 2005

1. Show that the limit below does exist. Hint: Consider a change of variables. There are also other ways of doing this . . .

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz}{x^2 + y^2 + z^2}$$

2. Does the following limit exist?

$$\lim_{(x,y,z,w) \rightarrow (0,0,0,0)} \frac{x^2 + y^2 + z^2 - w^2}{x^2 + y^2 + z^2 + w^2}$$

3. Use the laws of limits to find

$$\lim_{(x,y) \rightarrow (4,1)} \frac{x^2y - 16y \sin xy^2}{x - 4} \frac{1}{y^2}$$

4. Discuss the continuity of the function

$$h(x, y, z) = \begin{cases} \frac{\sin x^2 - y^2}{x^2 - y^2} & \text{unless } x^2 = y^2 \\ 1 & \text{if } x^2 = y^2 \end{cases}$$

5. Suppose $f_x(x, y) = y$ and $f_y(x, y) = x + y$. What is $f(x, y)$? What about $g_x(x, y) = x + 4y$ and $g_y(x, y) = 3x - y$?
6. What vector is normal to the surface $z = f(x, y) = 5x^2 + 7y^3 + 2x + 3y + 6$ at the point $(0, 0, 6)$?
7. Which of the following could be the equation of the tangent plane to the surface $z = x^2 + y^2$ at a point (a, b) in the first quadrant?
 - (a) $z = -3x + 4y + 7$
 - (b) $z = 2x - 4y + 5$
 - (c) $z = 6x + 6y - 18$
 - (d) $z = -4x - 4y + 24$

Warm-Up for Thursday

Let $f(x, y, z) = e^{xyz}$. Find f_x, f_{yx}, f_{zz} .