

## Merit Worksheet 14 - Math 242, Fall 2005

1. Find the one point at which the plane tangent to the surface

$$z = x^2 + 2xy + 2y^2 - 6x + 8y$$

is horizontal.

2. Let  $f(x, y, z, w) = w^2 \sin(x^2 + y^2 + z^2)$ . Find

$$f_{xyzw}(\sqrt{\pi/12}, \sqrt{\pi/12}, \sqrt{\pi/6}, 1) \quad \text{and} \quad \left. \frac{\partial^2 f}{\partial x \partial w} \right|_{0,0,\sqrt{\pi/4},2}.$$

3. Consider  $f(x, y, z) = x^2 \cos(y^3 + z^2)$ .
  - (a) Why do we know that  $f_{zyyxxx} = 0$  without doing any computations?
  - (b) Do we also know, without any computations, that  $f_{xyzzz} = 0$ ?
4. Suppose the function  $f(x, y)$  has continuous second partial derivatives and that

$$f_x(x, y) = axy^3 + 4e^x \sin 2y \quad \text{and} \quad f_y(x, y) = 3x^2y^2 + be^x \cos 2y + 1$$

Find  $f$ .

5. Find the maximum and minimum values attained by the function  $f(x, y) = x^2 + y^2 - x$  on the square with vertices  $(\pm 1, \pm 1)$ .
6. Find the maximum and minimum values attained by the function  $f(x, y) = xy^2$  on the circular disk  $x^2 + y^2 \leq 3$ .
7. Jose runs a lemonade stand. Into every gallon of lemonade he makes he pours  $x$  cups of lemon juice and  $y$  cups of sugar. The rest is water. He knows from past experience that if he adds less than two cups of (flavoured) ingredients it tastes too watery and if he adds more than four, it tastes too strong. Some recipes taste better than others and lead to more profit. His profit is given by  $P(x, y) = xy^2 - 6xy + 8x + 4$ . What are the most and least profitable recipes for Jose?

## Warm-up for Thursday

Read Section 12.5.

Find every point on the surface  $z = 2xye^{-1/8(4x^2+y^2)}$  at which the tangent plane is horizontal.

**Nothing to do with calculus** On a desert island, five students and a monkey gather coconuts all day, then sleep. The first student awakens and decides to gather her share. She divides the coconuts into 5 equal shares with one left over, gives the extra one to the monkey, hides her share, and goes to sleep. Later, the second student awakens and takes his fifth from the remaining pile. He, too, finds one extra and gives it to the monkey. Each of the three remaining students does likewise in turn. Find the minimum number of coconuts originally present.