Math 242 Merit Fall 2005 - Worksheet 15

1. Find the maximum and minimum values attained by the function $f(x, y) = xy^2$ on the circular disk $x^2 + y^2 \leq 3$.

2. Jose runs a lemonade stand. Into every gallon of lemonade he makes he pours x cups of lemon juice and y cups of sugar. The rest is water. He knows from past experience that if he adds less than two cups of (flavoured) ingredients it tastes too watery and if he adds more than four, it tastes too strong. Some recipes taste better than others and lead to more profit. His profit is given by $P(x, y) = xy^2 - 6xy + 8x + 4$. What are the most and least profitable recipes for Jose?

3. The function $f(x, y) = x^3 + 12xy + y^4$ has

   (a) No global maximum or minimum

   (b) A global max but no global min

   (c) A global min but no global max

   (d) Both a global max and a global min

4. Suppose that the function f is continuous on the disk D bounded by the unit circle $x^2 + y^2 = 1$. Is it possible that $f(x, y)$ attains both its maximum and its minimum values on D at points of the boundary circle? Illustrate your answer with an example.

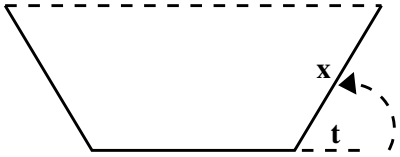
5. Consider $z = 2x^2 + 8xy + y^4$. Does this surface open upwards or downwards? What is its highest/lowest point?

6. Prof Hart's cardboard box factory has an order for open-topped boxes with a volume of 600 in^3. The material for the bottom of the box costs $6¢/in^2$ and the material for its sides costs $5¢/in^2$. What are the dimensions of the box that is most economical to manufacture?

7. Find the first octant point $P(x, y, z)$ on the plane $2x + 3y + z = 49$ which is closest to the point $Q(7, -7, 0)$.

8. Find the maximum possible product of three positive numbers whose sum is 120.

9. A very long rectangle of sheet metal has width L and is to be folded to make a rain gutter. Maximize its volume by maximizing the cross-sectional area, as shown.



Warm-Up for Tuesday

Read Section 12.6. Find the differential dw for the following:

1. $w = e^{-x^2-y^2}$
2. $w = xz^2 - yx^2 + zy^2$